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Homework 2 Solution Set

$\boxed{1.2}$ 4. $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}$ so $x_1 = -3$ $x_2 = 0$ $x_3 = 7$

b. From the matrix we can read off:

$$\begin{aligned} x_1 \quad\quad &+ -7x_4 = 8 \\ x_2 \quad &+ 3x_4 = 2 \\ x_3 &+ x_4 = -5 \end{aligned}$$

Here, we cannot solve definitely for x_4, so we will assign it an arbitrary value (T). This is because it is a free variable.

Then,

$$\begin{aligned} x_1 &= 7T + 8 \\ x_2 &= -3T + 2 \\ x_3 &= -T - 5 \\ x_4 &= T \end{aligned}$$

c. Here, x_2 and x_5 are free variables. We then assign them arbitrary values, s.t. (such that) $\boxed{x_2 = S}$ and $\boxed{x_5 = T}$

Then,

$$\begin{aligned} x_1 - 6x_2 \quad &+ 3x_5 = -2 \quad \text{so} \\ x_3 \quad &+ 4x_5 = 7 \quad \text{so} \\ x_4 &+ 5x_5 = 8 \quad \text{so} \end{aligned} \qquad \boxed{\begin{aligned} x_1 &= -2 - 3T + 6S \\ x_3 &= 7 - 4T \\ x_4 &= 8 - 5T \end{aligned}}$$

d. Here, the Third row of the matrix reads:

$$0x_1 + 0x_2 + 0x_3 = 1$$

$0 = 1$ is false, so this set of equations has no solution.

This Problem was worth 10 points.

5. We can see from te Third line: $\boxed{x_3 = 5}$

Sub into second line:

$1x_2 + 2x_3 = 2$ $\quad 1x_2 + 2 \cdot 5 = 2$ $\quad \boxed{x_2 = -8}$

again sub in and subtract st $\quad \boxed{x_1 = -37}$

b. Here, x_4 is a free variable, so let $\boxed{x_4 = T}$ Then, $x_3 + x_4 = 2$ so $\boxed{x_3 = 2 - T}$

$x_2 + 4x_3 + -9x_4 = 3$

$x_2 + 4(2-T) - 9(T) = 3$ $\quad x_2 = 3 + 9T - 8 + 4T$

$\boxed{x_2 = -5 + 13T}$

$x_1 + 8(x_3) + -5x_4 = 6$ $\quad x_1 + 8(2-T) - 5T = 6$

$x_1 = 6 - 16 + 8t + 5T$ $\quad \boxed{x_1 = -10 + 13T}$

5c. x_2 and x_5 are free variables:

Let $\boxed{x_2 = T}$ and $\boxed{x_5 = s}$

$x_4 + 3x_5 = 9$ $\quad \boxed{x_4 = 9 - 3s}$

$x_3 + x_4 + 6x_5 = 5$

$x_3 + 9 - 3s + 6s = 5$ $\quad \boxed{x_3 = -4 - 3s}$

$x_1 + 7x_2 - 2x_3 - 8x_5 = -3$

$x_1 + 7T - 2(-4 - 3s) - 8s = -3$

$x_1 = -3 - 8 - 7T - 6s + 8s = \boxed{-11 - 7T + 2s = x_1}$

d. The Third row reads: $0 = 1$ This has no Solution.

6. $\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$ we have our leading one in the 1st row, 1st column, so we must get 0's below it.

Let new Row 2 = Row 2 + Row 1

Let new Row 3 = Row 3 − 3Row 1

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix} \times -1 = \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{bmatrix} + 10 \text{ row } 2$$

$$= \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{bmatrix} \times -\frac{1}{52} = \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix} + 5 \text{ row } 3 = \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} - \text{Row2} - 2\,\text{Row3} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$\Rightarrow x_1 = 3 \qquad x_2 = 1 \qquad x_3 = 2$

6. b. $$\begin{bmatrix} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix} \times \frac{1}{2} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix} \begin{array}{l} \\ +2\,\text{row1} \\ -8\,\text{row1} \end{array} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{bmatrix}$$

Notice here that rows 2 and 3 are redundant → if we add row 2 to row three, and divide row 2 by 7

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \text{row 2} = \begin{bmatrix} 1 & 0 & 3/7 & -1/7 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x_3 is a free variable, let $x_3 = s$

$x_1 = -1/7 - 3/7 s \qquad x_2 = 1/7 - 4/7 s$

6c $\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix} \begin{matrix} \\ -2\text{ row}1 \\ +\text{ row}1 \\ -3\text{ row}1 \end{matrix} = \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \times\frac{1}{3} \\ \\ \\ \end{matrix}$

$= \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ -\text{ row }2 \\ -3\text{ row }2 \end{matrix} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} +\text{ row }2 \\ \\ \\ \\ \end{matrix}$

$= \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ so x_3 and x_4 are free.

$x_3 = s \qquad x_4 = T$

$x_1 = T - 1 \qquad x_2 = 2s$

d. $\begin{bmatrix} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{bmatrix} \begin{matrix} \times\frac{1}{3} \\ \\ \\ \end{matrix}$

we cannot have a zero in A_{11} so we switch

$\begin{bmatrix} 1 & 2 & -1 & -\frac{2}{3} \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{bmatrix} \begin{matrix} \\ \\ -6\times\text{row}1 \end{matrix} = \begin{bmatrix} 1 & 2 & -3 & -\frac{2}{3} \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & -3 & -\frac{2}{3} \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{bmatrix} + -3 \text{ row } 2 = \begin{bmatrix} 1 & 2 & -3 & -\frac{2}{3} \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

row 3 reads $0 = 6$. <u>Inconsistent</u>

7a. we had: $\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

so $x_3 = 2$

$x_2 - 5x_3 = -9 \qquad x_2 = -9 + 10 = 1$

$x_1 + x_2 + 2x_3 = 8$ st

$x_1 + 1 + 4 = 8$ st $x_1 = 3$

b. We had $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$ we still Let $x_3 = s$

and $x_2 = \frac{1}{7} - \frac{4}{7}s.$

$x_1 + x_2 + x_3 = 0 \qquad x_1 = -\left(\frac{1}{7} - \frac{4}{7}s\right) - s$

$= -\frac{1}{7} - \frac{3}{7}s$

7a We had: $\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $x_3 = s$ $x_4 = T$

$x_2 - 2x_3 = 0$ $x_2 = 2s$

$x_1 - x_2 + 2x_3 - x_4 = -1$

$x_1 - 2s + 2s + -T = -1$ $x_1 = -1 + T$

7d is still inconsistent.

12. This has 4 unknowns and only 3 equations. It therefore cannot be determined and must have a non-trivial solution. (In fact, it must have infinitly many of them.) Compare Theorem 1.2.1 from page 18.

So, yes.

b. No. The 3rd equation means $x_3 = 0$, we can use this to see that $x_2 = 0$, and this shows that $x_1 = 0$

c. Yes, again. Theorem 1.2.1

d. Here, equation 2 = 2 x equation 1. So, we only really have 1 equation and 2 unknowns. So yes

13. I like Gauss-Jordan in general

a. $\begin{bmatrix} 2 & 1 & 3 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \curvearrowright \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} -2\text{ row}1 = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \times -\frac{1}{3}$

$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & +1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} - \text{row } 2 = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \times \frac{1}{2} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \text{row } 3$

$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} -2\text{ row } 2 \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad X_1 = x_2 = x_3 = 0$

b. $\begin{bmatrix} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix} \times \frac{1}{3} = \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix} -5\text{ row}1$

$\begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{8}{3} & -\frac{2}{3} & -\frac{8}{3} & 0 \end{bmatrix} \times -\frac{3}{8} = \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{4} & 1 & 0 \end{bmatrix}$

$x_3 = s \quad X_4 = T \quad x_2 = -\frac{1}{4}s - t$

$x_1 = \frac{1}{3}\left(\frac{1}{4}s + t\right) - \frac{1}{3}s + -\frac{1}{3}t$

$x_1 = -\frac{1}{3}s + \frac{1}{12}s = -\frac{1}{4}s$

13c $\begin{bmatrix} 0 & 2 & 2 & 4 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{bmatrix} \curvearrowright \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{bmatrix} \begin{array}{l} \\ \\ -2\text{ row 1} \\ +2\text{ row 1} \end{array} = \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{bmatrix} \begin{array}{l} \\ \times\frac{1}{2} \\ \\ \\ \end{array}$

$$\begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{bmatrix} \begin{array}{l} \\ \\ -3\text{ row 2} \\ -\text{row 2} \end{array} = \begin{bmatrix} 1 & 0 & -\cancel{1} & 3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -10 & 0 \end{bmatrix}$$

$x_4 = 0$ let $x_3 = s$ $x_2 = -s$ $x_1 = s$

25. If we plug the four points into the equations we get:

$$d = 10$$

$$a + b + c + d = 7$$
$$27a + 9b + 3c + d = -11$$
$$64a + 16b + 4c + d = -14$$

We have $d = 10$, so eliminate it from the equations.

$$a + b + c = -3$$
$$27a + 9b + 3c = -21$$
$$64a + 16b + 4c = -24$$

$$\begin{bmatrix} 1 & 1 & 1 & -3 \\ 27 & 9 & 3 & -21 \\ 64 & 16 & 4 & -24 \end{bmatrix} \begin{array}{l} \\ -27\text{ row 1} \\ -64\text{ row 2} \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & -18 & -24 & 60 \\ 0 & -48 & -60 & 168 \end{bmatrix} \times -\tfrac{1}{18} = \begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & 1 & 4/3 & -\frac{10}{3} \\ 0 & -48 & -60 & 168 \end{bmatrix} \begin{array}{l} \\ \\ +48\text{ row 2} \end{array} = \begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & 1 & 4/3 & -\frac{10}{3} \\ 0 & 0 & 4 & 8 \end{bmatrix} \begin{array}{l} \\ \\ \times\frac{1}{4} \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & 1 & 4/3 & -10/3 \\ 0 & 0 & 1 & 2 \end{bmatrix} -4/3 \times \text{row}3 = \begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \end{bmatrix} -\text{row }2 -\text{row }3$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$a = 1 \quad b = -6 \quad c = 2$

and d still $= 10$

27. I will prove in one set of operations both a) and b).

$ax + by = k$, $cx + dy = l$ $\Rightarrow \begin{bmatrix} a & b & k \\ c & d & e \end{bmatrix}$ either $a \neq 0$ or $c \neq 0$, else $ad - bc = 0$, which is false.

Without loss of generality, we will let $a \neq 0$

$$\begin{bmatrix} a & b & k \\ c & d & e \end{bmatrix} \times 1/a = \begin{bmatrix} 1 & b/a & k/a \\ c & d & e \end{bmatrix} - c \cdot \text{row}1$$

$$\begin{bmatrix} 1 & b/a & k/a \\ 0 & d - \frac{cb}{a} & e - \frac{ck}{a} \end{bmatrix}$$

$d - \frac{cb}{a} \neq 0$ because $ad - bc \neq 0$

so divide row 2 by $d - \frac{cb}{a}$

$$\begin{bmatrix} 1 & b/a & k/a \\ 0 & 1 & \left(e - \frac{ck}{a}\right) \cdot \left(d - \frac{cb}{a}\right)^{-1} \end{bmatrix} - b/a \text{ row}2$$

$$\begin{bmatrix} 1 & 0 & k/a - b/a\left(e - \frac{ck}{a}\right)\left(d - \frac{cb}{a}\right)^{-1} \\ 0 & 1 & \left(e - \frac{ck}{a}\right)\left(d - \frac{cb}{a}\right)^{-1} \end{bmatrix}$$

The key to all those steps was that they were all allowable because we never divided by zero, as we could prove because $ad - bc \neq 0$. We will see this expression later and learn more about its importance

1.3 6, $(2D^T - E)A$

$$\left(2 \cdot \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}^T - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}\right) \cdot \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 2 & -2 & 6 \\ 10 & 0 & 4 \\ 4 & 2 & 8 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \cdot \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}\right)$$

$$= \begin{bmatrix} -4 & -3 & 3 \\ 11 & -1 & 2 \\ 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

= M_{11}, for example, is 1st row x 1st column =

$-4 \cdot 3 + -3 \cdot -1 + 3 \cdot 1 = -12 + 3 + 3 = -6$

So that the whole matrix =

$$\begin{bmatrix} -6 & -3 \\ 36 & 0 \\ 4 & 7 \end{bmatrix}$$

1.3 6.b. Here, let us compare the sizes of the matricies we are multiplying:

$$2\times2 \cdot 2\times3 + 2\times2$$
$$= 2\times3 + 2\times2 \quad \text{is undefined.}$$

c. $(-AC)^T + 5D^T$

$$\left(-\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}\right)^T + 5\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -12 & -6 \\ -5 & 2 & -8 \\ -4 & -5 & -7 \end{bmatrix}^T + \begin{bmatrix} 5 & 25 & 10 \\ -5 & 0 & 5 \\ 15 & 10 & 20 \end{bmatrix}^T$$

$$\begin{bmatrix} -3 & -5 & -4 \\ -12 & 2 & -5 \\ -6 & -8 & -7 \end{bmatrix} + \begin{bmatrix} 5 & -5 & \overset{15}{\cancel{10}} \\ 25 & 0 & 10 \\ 10 & 5 & 20 \end{bmatrix} = \begin{bmatrix} 2 & -10 & 11 \\ 13 & 2 & 5 \\ 4 & -3 & 13 \end{bmatrix}$$

6d. $(CBA^T - 2C)^T$

$$\left(\begin{bmatrix}4&-1\\0&2\end{bmatrix}\begin{bmatrix}3&-1&1\\0&2&1\end{bmatrix} - 2\begin{bmatrix}1&4&2\\3&1&5\end{bmatrix}\right)^T$$

$$=\left(\begin{bmatrix}12&-6&3\\0&4&2\end{bmatrix}-\begin{bmatrix}2&8&4\\6&2&10\end{bmatrix}\right)^T$$

$$=\begin{bmatrix}10&-14&-1\\-6&2&-8\end{bmatrix}^T = \begin{bmatrix}10&-6\\-14&2\\-1&-8\end{bmatrix}$$

e. $B^T(CC^T - A^TA)$

$$\begin{bmatrix}4&0\\-1&2\end{bmatrix}\left(\begin{bmatrix}1&4&2\\3&1&5\end{bmatrix}\begin{bmatrix}1&3\\4&1\\2&5\end{bmatrix} - \begin{bmatrix}3&-1&1\\0&2&1\end{bmatrix}\begin{bmatrix}3&0\\-1&2\\1&1\end{bmatrix}\right)$$

$$\begin{bmatrix}4&0\\-1&2\end{bmatrix}\left(\begin{bmatrix}21&17\\17&35\end{bmatrix}-\begin{bmatrix}11&-1\\-1&5\end{bmatrix}\right)$$

$$=\begin{bmatrix}4&0\\-1&2\end{bmatrix}\begin{bmatrix}10&18\\18&30\end{bmatrix} = \begin{bmatrix}40&72\\26&42\end{bmatrix}$$

6f. $D^T E^T - (ED)^T$

$$\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} - \left(\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 14 & 4 & 12 \\ 36 & -1 & 26 \\ 25 & 7 & 21 \end{bmatrix} - \begin{bmatrix} 14 & 36 & 25 \\ 4 & -1 & 7 \\ 12 & 26 & 21 \end{bmatrix}^T$$

$$= \begin{bmatrix} 14 & 4 & 12 \\ 36 & -1 & 26 \\ 25 & 7 & 21 \end{bmatrix} - \begin{bmatrix} 14 & 4 & 12 \\ 36 & -1 & 26 \\ 25 & 7 & 21 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

7. jth column of AB = [A][Jth column of B]
ith row of AB = [ith row of A][B]

a = [1st row of A][B]

$$= \begin{bmatrix} 3 & -2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 67 & 41 & 41 \end{bmatrix}$$

b. [3rd row of A][B]

$$\begin{bmatrix} 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 63 & 67 & 57 \end{bmatrix}$$

c. [A][2nd column of B]

$$\begin{bmatrix} 3 & -2 & 7 \\ 6 & 4 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix}$$

d. [B][1st column of A]

$$\begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 63 \end{bmatrix}$$

12. Let A be a $r \times c$ matrix and B is a $s \times d$ matrix

AB is $\cdot$ $r \times c \cdot s \times d$ so $c = s$
and BA is $s \times d \cdot r \times c$ so $d = r$
Because to be defining, must be in form $a \times b \cdot b \cdot c$.
AB is a $r \times d$, which is square, b/c $d = r$
BA is a $s \times c$, which is square, b/c $c = s$

~~13.~~
b. Let B be a $x \times y$ matrix
we know
$m \times n \cdot x \times y \cdot m \times m$ is defined, so
$x = n$ $y = m$ so B is a
$n \times m$ matrix.

13.
$$\begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$$

b
$$\begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$$

e. [third row of A] [A]

$$\begin{bmatrix} 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 24 & 56 & 97 \end{bmatrix}$$

f. [A] [3rd column of A]

$$\begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix} = \begin{bmatrix} 76 \\ 98 \\ 97 \end{bmatrix}$$

g. $$\begin{bmatrix} y_1 & \cdots & y_m \end{bmatrix} \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

Multiply 1st row + 1st column, etc to get:

$$\begin{bmatrix} y_1 a_{11} + \cdots + y_m a_{m1} & \cdots & y_1 a_{1n} + \cdots + y_m a_{mn} \end{bmatrix}$$

we can write

$$y_1 \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix} + \cdots + y_m \begin{bmatrix} a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

which is what is desired.

23. Claim: If A and B are $n \times n$ matrices, then $tr(A+B) = tr(A) + tr(B)$

Pf.

Let $A = \begin{bmatrix} a_{11} & & \\ & \ddots & \\ & & a_{nn} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & & \\ & \ddots & \\ & & b_{nn} \end{bmatrix}$

$$tr(A+B) = tr \begin{bmatrix} a_{11}+b_{11} & & \\ & \ddots & \\ & & a_{nn}+b_{nn} \end{bmatrix} = a_{11}+b_{11} + \cdots + a_{nn}+b_{nn}$$

$$= a_{11} + \cdots + a_{nn} + b_{11} + \cdots + b_{nn} = tr \begin{bmatrix} a_{11} & \\ & \ddots \; a_{nn} \end{bmatrix} + tr \begin{bmatrix} b_{11} & \\ & \ddots \; b_{nn} \end{bmatrix}$$

$$= tr(A) + tr(B)$$

$\blacksquare$

11.4

2.

$$\begin{bmatrix} 53 & 96 & 37 \\ 47 & 87 & 41 \\ 60 & 92 & 36 \end{bmatrix} \Rightarrow \begin{bmatrix} 16 & 59 & 0 \\ 6 & 46 & 0 \\ 24 & 56 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 10 & 13 & 0 \\ 0 & 0 & 0 \\ 18 & 10 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 & 13 & 0 \\ 0 & 0 & 0 \\ 18 & 10 & 0 \end{bmatrix}$$

too few lines, subtract 10 from all uncovered, add it to all double covered.

$$\begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 10 \\ 8 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 10 \\ 8 & 0 & 0 \end{bmatrix}$$ optimal

$$\begin{bmatrix} \boxed{0} & 3 & 0 \\ 0 & \boxed{0} & 10 \\ 8 & 0 & \boxed{0} \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 3 & \boxed{0} \\ \boxed{0} & 0 & 10 \\ 8 & \boxed{0} & 0 \end{bmatrix}$$

Total cost $= 176$

4.

$$\begin{bmatrix} 0 & 3 & 0 & 4 & \boxed{0} \\ 4 & \boxed{0} & 6 & 1 & 5 \\ 3 & 1 & \boxed{0} & 4 & 0 \\ \boxed{0} & 1 & 6 & 2 & 1 \\ 0 & 0 & 0 & \boxed{0} & 3 \end{bmatrix}$$

or

$$\begin{bmatrix} 7 & 4 & 7 & 3 & \boxed{10} \\ 5 & \boxed{9} & 3 & 8 & 7 \\ 3 & 5 & \boxed{6} & 2 & 9 \\ \boxed{6} & 5 & 0 & 4 & 8 \\ 0 & 0 & 0 & \boxed{0} & 0 \end{bmatrix} \begin{aligned} &= 10 + 9 + 6 + 6 + 0 \\ &= 19 + 12 = \boxed{31} \end{aligned}$$

compare to old solution:

$$\begin{bmatrix} 7 & 4 & \boxed{7} & 3 & 10 \\ 5 & \boxed{9} & 3 & 8 & 7 \\ 3 & 5 & 6 & 2 & \boxed{9} \\ \boxed{6} & 5 & 0 & 4 & 8 \\ 0 & 0 & 0 & \boxed{0} & 0 \end{bmatrix} \begin{aligned} &7 + 9 + 9 + 6 + 0 \\ &16 + 15 = \boxed{31} \end{aligned}$$

5a. $\begin{bmatrix} -150 & -65 & -210 & -135 & 0 \\ -175 & -75 & -230 & -155 & 0 \\ -135 & -85 & -200 & -140 & 0 \\ -140 & -70 & -190 & -130 & 0 \\ -170 & -50 & -200 & -160 & 0 \end{bmatrix} \Rightarrow$

The dealer wants to maximize the profit, so the "costs" are negative to him!

$\begin{bmatrix} 60 & 145 & 0 & 75 & 210 \\ 55 & 155 & 0 & 75 & 230 \\ 65 & 115 & 0 & 60 & 200 \\ 50 & 120 & 0 & 60 & 190 \\ 30 & 150 & 0 & 40 & 200 \end{bmatrix} \Rightarrow$

$\begin{bmatrix} 30 & 30 & 0 & 35 & 20 \\ 25 & 40 & 0 & 35 & 40 \\ 35 & 0 & 0 & 20 & 10 \\ 20 & 5 & 0 & 20 & 0 \\ 0 & 35 & 0 & 0 & 10 \end{bmatrix}$

not enough lines.
not optimal
Subtract 20 from uncovered, add to double covered

$\begin{bmatrix} 10 & 10 & 0 & 15 & 0 \\ 5 & 20 & 0 & 15 & 20 \\ 35 & 0 & 20 & 20 & 10 \\ 20 & 5 & 20 & 20 & \boxed{0} \\ \boxed{0} & 35 & 20 & 0 & 10 \end{bmatrix}$

Subtract 5 from uncovered, add 5 to double covered

$$\begin{bmatrix} 5 & 10 & \boxed{0} & 10 & 0 \\ \boxed{0} & 20 & 0 & 10 & 20 \\ 30 & \boxed{0} & 20 & 20 & 10 \\ 15 & 5 & 20 & 15 & \boxed{0} \\ 0 & 40 & 25 & \boxed{0} & 15 \end{bmatrix}$$

Biddr	1	to 3	=	210
	2	1	=	175
	3	2	=	85
	4	"coin 5"	=	0
	5	4	=	160

Total = 630