HW6. Jon Playfurth

11.6

20 1. $n=1$

$$x_1 = \begin{bmatrix} .4 & .5 \\ .6 & .5 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} .4 \\ .6 \end{bmatrix} \qquad x_2 = \begin{bmatrix} .4 & .5 \\ .6 & .5 \end{bmatrix}\begin{bmatrix} .4 \\ .6 \end{bmatrix} = \begin{bmatrix} .46 \\ .54 \end{bmatrix}$$

$$x_3 = P \cdot \begin{bmatrix} .46 \\ .54 \end{bmatrix} = \begin{bmatrix} .454 \\ .546 \end{bmatrix} \qquad x_4 = P \cdot \begin{bmatrix} .454 \\ .546 \end{bmatrix} = \begin{bmatrix} .4546 \\ .5454 \end{bmatrix} \qquad x_5 = P x_4 = \begin{bmatrix} .45454 \\ .54546 \end{bmatrix}$$

b. It needs to have some power where all entries are > 0, and P^1 satisfies this.

Start state: ~~Sol~~

We know

$$P q = q \qquad Pq - q = 0 \quad \text{or}$$

$$q - Pq = 0 \qquad (I - P) q = 0$$

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .4 & .5 \\ .6 & .5 \end{bmatrix}\right)\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} .6 & -.5 \\ -.6 & .5 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \text{add the rows}$$

$$\left[\begin{array}{cc|c} .6 & -.5 & 0 \\ 0 & 0. & 0 \end{array}\right] \qquad .6q - .5q_2 = 0$$

$$.6 q_1 = .5 q_2 \qquad = c\begin{bmatrix} q_1 \\ 1.2 q_1 \end{bmatrix}$$
$$1.2 q_1 = q_2$$

$$q_1 + 1.2 q_1 = 1 \qquad 2.2 q_1 = 1 \qquad q_1 \tfrac{1}{2.2} = \tfrac{5}{11}$$

$$q = \begin{bmatrix} 5/11 \\ 6/11 \end{bmatrix}$$

7.

$$\begin{array}{ccc} \text{happy} & \text{sad} & \\ \text{day 1} & & \\ \left[\begin{matrix} 4/5 & 2/3 \\ 1/5 & 1/3 \end{matrix}\right] & & \begin{matrix} H \\ S \end{matrix} \end{array}$$ day 2 $= P$

$(I-P)q = 0$

$$\begin{bmatrix} 1/5 & -2/3 \\ -1/5 & 2/3 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$\frac{1}{5}q_1 - \frac{2}{3}q_2 = 0$

$\frac{1}{5}q_1 = \frac{2}{3}q_2$

$\frac{3}{10}q_1 = q_2$ $\qquad c\begin{bmatrix} q_1 \\ \frac{3}{10}q_1 \end{bmatrix}$ $\qquad c = \frac{10}{13}$

$\begin{bmatrix} 10/13 \\ 3/13 \end{bmatrix}$ He is happy $\boxed{10/13}$ of the time

8.

$$\begin{array}{c|ccc|c} & \multicolumn{3}{c}{\text{Period 1}} & \\ \text{reason} & 1 & 2 & 3 & \\ & .9 & .15 & .1 & 1 \\ & .05 & .75 & .05 & 2 \\ & .05 & .1 & .85 & 3 \end{array}$$ Period 2

$(I-P)q = 0$

$$\begin{bmatrix} .1 & -.15 & -.1 \\ -.05 & .25 & -.05 \\ -.05 & -.1 & .15 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1.5 & -1 \\ 0 & .175 & -.1 \\ 0 & -.175 & .1 \end{bmatrix}$$

$q_1 - 1.5q_2 = q_3$

$.175q_2 = .1q_3$

$1.75q_2 = q_3$ $\qquad q_2 = \frac{1}{1.75}q_3$

$q_1 = 1.5q_2 + 1.75q_2 = 3.25q_2$

$$c\begin{bmatrix} 3.25q_2 \\ q_2 \\ 1.75q_2 \end{bmatrix}$$

$c = 1/6$ $\quad q = \begin{bmatrix} 54.1\bar{6}\,\% \\ 16.6\,\% \\ 29.1\bar{6}\,\% \end{bmatrix}$

2.1

a	5
b	9
c	6
d	10
e	0
f	2

6 $\begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix}$ $\quad ad - bc = \sqrt{2}\sqrt{3} - 4\sqrt{6}$

$= \sqrt{6} - 4\sqrt{6} = \boxed{-3\sqrt{6}}$

8 $\begin{array}{ccccc} + & + & + & - & - \; - \\ \end{array}$

$\begin{bmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{bmatrix} \begin{matrix} -2 & 7 \\ 5 & 1 \\ 3 & 8 \end{matrix} = 190 - 190 = 0$

10. $\begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{bmatrix} \begin{matrix} -1 & 1 \\ 3 & 0 \\ 1 & 7 \end{matrix} = 37 - 41 = -4$

12. $\begin{bmatrix} c & -4 & 3 \\ 2 & 1 & c^2 \\ 4 & c-1 & 2 \end{bmatrix} \begin{matrix} c & -4 \\ 2 & 1 \\ 4 & c-1 \end{matrix}$

$= (-16x^2 + 8x - 6) - (x^4 - x^3 - 4)$

$= -x^4 + x^3 - 16x^2 + 8x - 2$

13. $ad - bc = 0$

20

$(x-2)(x+4) + 5 = 0$

$x^2 + 2x - 3 = 0$

$(x-1)(x+3) = 0$

$x = 1$ or 3

b. $\det = (x-4)(x^2 - x - 6)$

$= (x-4)(x-3)(x+2) = 0$

$x = 4$ or 3 or -2

17. The only permut is (54321) = 120
This is an even permutation, so multiply by +1

(54321)
(14325)
(12345)

so det = $\boxed{120}$

b. This Permutation: (15342)
(12345) so odd.

$= (-1) \cdot (120) = \boxed{-120}$