Section 2.4

2.c) $A=\begin{bmatrix}4 & -1 & 1 & 6\\ \cancel{0} & \cancel{0} & \cancel{-3} & \cancel{3}\\ 4 & 1 & 0 & 14\\ 4 & 1 & 3 & 2\end{bmatrix}$ $M_{22}=\begin{vmatrix}4 & 1 & 6\\ 4 & 0 & 14\\ 4 & 3 & 2\end{vmatrix} = \begin{vmatrix}4 & 1 & 6\\ 0 & -1 & 8\\ 0 & 2 & -4\end{vmatrix} = 4\begin{vmatrix}-1 & 8\\ 2 & -4\end{vmatrix} = -48$

$C_{22}=(-1)^4\cdot(-48) = -48$

d) $A=\begin{bmatrix}4 & -1 & 1 & 6\\ \cancel{0} & \cancel{0} & \cancel{-3} & \cancel{3}\\ 4 & 1 & 0 & 14\\ 4 & 1 & 3 & 2\end{bmatrix}$ $M_{21}=\begin{vmatrix}-1 & 1 & 6\\ 1 & 0 & 14\\ 1 & 3 & 2\end{vmatrix} = \begin{vmatrix}-1 & 1 & 6\\ 0 & 1 & 20\\ 0 & 4 & 8\end{vmatrix} = (-1)\begin{vmatrix}1 & 20\\ 4 & 8\end{vmatrix} = 72$

$C_{21}=(-1)^3(72) = -72$

3.d) $A=\begin{bmatrix}1 & -2 & 3\\ 6 & 7 & -1\\ -3 & 1 & 4\end{bmatrix}$ $\det A = -(-2)\begin{vmatrix}6 & -1\\ -3 & 4\end{vmatrix} + 7\begin{vmatrix}1 & 3\\ -3 & 4\end{vmatrix} - (1)\begin{vmatrix}1 & 3\\ 6 & -1\end{vmatrix} = 152$

e) $\det A = -3\begin{vmatrix}-2 & 3\\ 7 & -1\end{vmatrix} - \begin{vmatrix}1 & 3\\ 6 & -1\end{vmatrix} + 4\begin{vmatrix}1 & -2\\ 6 & 7\end{vmatrix} = 152$

f) $\det A = 3\begin{vmatrix}6 & 7\\ -3 & 1\end{vmatrix} - (-1)\begin{vmatrix}1 & -2\\ -3 & 1\end{vmatrix} + 4\begin{vmatrix}1 & -2\\ 6 & 7\end{vmatrix} = 152$

4. adj A

a) $A=\begin{bmatrix}1 & -2 & 3\\ 6 & 7 & -1\\ -3 & 1 & 4\end{bmatrix}$

$C_{11}=\begin{vmatrix}7 & -1\\ 1 & 4\end{vmatrix}=29$ $C_{12}=-\begin{vmatrix}6 & -1\\ -3 & 4\end{vmatrix}=-21$ $C_{13}=\begin{vmatrix}6 & 7\\ -3 & 1\end{vmatrix}=27$

$C_{21}=-\begin{vmatrix}-2 & 3\\ 1 & 4\end{vmatrix}=11$ $C_{22}=\begin{vmatrix}1 & 3\\ -3 & 4\end{vmatrix}=13$ $C_{23}=-\begin{vmatrix}1 & -2\\ -3 & 1\end{vmatrix}=5$

$C_{31}=\begin{vmatrix}-2 & 3\\ 7 & -1\end{vmatrix}=-19$ $C_{32}=-\begin{vmatrix}1 & 3\\ 6 & -1\end{vmatrix}=19$ $C_{33}=\begin{vmatrix}1 & -2\\ 6 & 7\end{vmatrix}=11$

$\text{adj}(\;)=\begin{bmatrix}29 & 11 & -19\\ -21 & 13 & 19\\ 27 & 5 & 19\end{bmatrix}$

b) $A^{-1}=?$

$\det(A)=152$ (from problem 3)

$A^{-1}=\frac{1}{\det(A)}\text{adj}(A) = \begin{bmatrix}29/152 & 11/152 & -19/152\\ -21/152 & 13/152 & 19/152\\ 27/152 & 5/152 & 19/152\end{bmatrix}$

6. $\begin{vmatrix}3 & 3 & 1\\ 1 & 0 & -4\\ 1 & -3 & 5\end{vmatrix} = 3\begin{vmatrix}0 & -4\\ -3 & 5\end{vmatrix} - (1)\begin{vmatrix}3 & 1\\ -3 & 5\end{vmatrix} + (1)\begin{vmatrix}3 & 1\\ 0 & -4\end{vmatrix} = -36 - 18 + (-12) = \underline{\underline{-66}}$

8. $\begin{vmatrix} k+1 & k-1 & 7 \\ 2 & k-3 & 4 \\ 5 & k+1 & k \end{vmatrix} = (k+1)\begin{vmatrix} k-3 & 4 \\ k+1 & k \end{vmatrix} - 2\begin{vmatrix} k-1 & 7 \\ k+1 & k \end{vmatrix} + 5\begin{vmatrix} k-1 & 7 \\ k-3 & 4 \end{vmatrix}$

$= (k+1)\left[k^2-3k-(4k+4)\right] - 2\left[(k^2-k)-(7k+7)\right] +$
$5\left[(4k-4)-(7k-21)\right] = k^3-8k^2-10k+95$

12. $\begin{vmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{vmatrix}$ $\quad det(A) = 3\begin{vmatrix} 2 & 3 \\ -2 & -4 \end{vmatrix} = -6$

$C_{11} = \begin{vmatrix} 3 & 2 \\ 0 & -4 \end{vmatrix} = -12 \quad C_{21} = -\begin{vmatrix} 0 & 3 \\ 0 & -4 \end{vmatrix} = 0 \quad C_{31} = \begin{vmatrix} 0 & 3 \\ 3 & 2 \end{vmatrix} = -9$

$C_{12} = -\begin{vmatrix} 0 & 2 \\ -2 & -4 \end{vmatrix} = -4 \quad C_{22} = \begin{vmatrix} 2 & 3 \\ -2 & -4 \end{vmatrix} = -2 \quad C_{32} = -\begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} = -4$

$C_{13} = \begin{vmatrix} 0 & 3 \\ -2 & 0 \end{vmatrix} = 6 \quad C_{23} = -\begin{vmatrix} 2 & 0 \\ -2 & 0 \end{vmatrix} = 0 \quad C_{33} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6$

$adj\,A = \begin{bmatrix} -12 & 0 & -9 \\ -4 & -2 & -4 \\ 6 & 0 & 6 \end{bmatrix} \qquad A^{-1} = \frac{1}{det(A)} adj(A) = \begin{bmatrix} 2 & 0 & 3/2 \\ 2/3 & 1/3 & 2/3 \\ -1 & 0 & -1 \end{bmatrix}$

16. $7x_1 - 2x_2 = 3$
$3x_1 + x_2 = 5$

$A = \begin{bmatrix} 7 & -2 \\ 3 & 1 \end{bmatrix} \quad A_1 = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 7 & 3 \\ 3 & 5 \end{bmatrix}$

$det(A) = 13 \quad det(A_1) = 13 \quad det(A_2) = 26$

$x_1 = \frac{det(A_1)}{det(A)} = 13/13 = 1 \qquad x_2 = \frac{det(A_2)}{det(A)} = 26/13 = 2$

23. $\begin{vmatrix} 4 & 1 & 1 & 1 \\ 3 & 7 & -1 & 1 \\ 7 & 3 & -5 & 8 \\ 1 & 1 & 1 & 2 \end{vmatrix} = -\begin{vmatrix} 1 & 1 & 1 & 2 \\ 3 & 7 & -1 & 1 \\ 7 & 3 & -5 & 8 \\ 4 & 1 & 1 & 1 \end{vmatrix} = \begin{matrix} R_2-3R_1 \\ R_3-7R_1 \\ R_4-4R_1 \end{matrix} - \begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & 4 & -4 & -5 \\ 0 & -4 & -12 & -6 \\ 0 & -3 & -3 & -7 \end{vmatrix} =$

$-4\begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -5/4 \\ 0 & -4 & -12 & -6 \\ 0 & -3 & -3 & -7 \end{vmatrix} = -4\begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -5/4 \\ 0 & 0 & -16 & -11 \\ 0 & 0 & -6 & -43/4 \end{vmatrix} = 64\begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -5/4 \\ 0 & 0 & 1 & 11/16 \\ 0 & 0 & -6 & -43/4 \end{vmatrix} =$

$64\begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -5/4 \\ 0 & 0 & 1 & 11/16 \\ 0 & 0 & 0 & -106/16 \end{vmatrix} = (64)(1)(1)(-106/16) = -424$

$$\det(A_y) = \begin{vmatrix} 4 & 6 & 1 & 1 \\ 3 & 1 & -1 & 1 \\ 7 & -3 & -5 & 8 \\ 1 & 3 & 1 & 2 \end{vmatrix} = - \begin{vmatrix} 1 & 3 & 1 & 2 \\ 3 & 1 & -1 & 1 \\ 7 & -3 & -5 & 8 \\ 4 & 6 & 1 & 1 \end{vmatrix} =$$

$$- \begin{vmatrix} 1 & 3 & 1 & 2 \\ 0 & -8 & -4 & -5 \\ 0 & -24 & -12 & -6 \\ 0 & -6 & -3 & -7 \end{vmatrix} = (-1)(-6)(-8) \begin{vmatrix} 1 & 3 & 1 & 2 \\ 0 & 1 & 1/2 & 5/8 \\ 0 & 4 & 2 & 1 \\ 0 & -6 & -3 & -7 \end{vmatrix} = -48 \begin{vmatrix} 1 & 3 & 1 & 2 \\ 0 & 1 & 1/2 & 5/8 \\ 0 & 0 & 0 & -3/2 \\ 0 & 0 & 0 & 8 \end{vmatrix}$$

$$= (-48)(1)(1)(0)(8) = 0$$

$$y = \frac{\det(A_y)}{\det(A)} = 0$$