11.9

1. $(I - E)p = 0$

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1/2 & 1/3 \\ 1/2 & 2/3 \end{bmatrix}\right)\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & -1/3 \\ -1/2 & 1/3 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = 0 \qquad \tfrac{1}{2}p_1 - \tfrac{1}{3}p_2 = 0$$

$p_1 = 2 \quad p_2 = 3$

$$p = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

b. $$\begin{bmatrix} 1/2 & 0 & -1/2 \\ -1/3 & 1 & -1/2 \\ -1/6 & -1 & 1 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \tfrac{1}{2}p_1 - \tfrac{1}{2}p_3 = 0 \qquad p_1 = p_3$$

$-\tfrac{1}{6}p_1 + p_3 = p_2$

Note: third equation is redundant.

$p_1 = p_3 = 6$

$-1 + 6 = p_2 \qquad p_2 = 5$

$p_1 = 6$
$p_2 = 5$
$p_3 = 6$

c. $$\begin{bmatrix} .65 & -.5 & -.3 \\ -.25 & .8 & -.3 \\ -.4 & -.3 & .6 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 65 & -50 & -30 \\ -25 & 80 & -30 \\ -40 & -30 & 60 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = 0 \qquad E_q\,1 - E_q\,2$$

$90p_1 - 130p_2 = 0$

multiply by 6 or,

let $\boxed{p_1 = 13 \quad p_2 = 9}$

$E_{q3} = -52 - 27 + 6p_3 = 0 \quad -79 = -6p_3 \qquad \boxed{p_3 = \frac{79}{6}}$

$\boxed{p_1 = 78 \quad p_2 = 54 \quad p_3 = 79}$

3. Need to show that E^m has all positive entries

$$E^2 = \begin{bmatrix} 0 & .2 & .5 \\ 1 & .2 & .5 \\ 0 & .6 & 0 \end{bmatrix} \begin{bmatrix} 0 & .2 & .5 \\ 1 & .2 & .5 \\ 0 & .6 & 0 \end{bmatrix} = \begin{bmatrix} .2 & .34 & .1 \\ .2 & .54 & .6 \\ .6 & .12 & .3 \end{bmatrix}$$

Here E^2 has all positive entries, so Thrm. Applies.

4. Grown

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} \text{consumed} \\ A \\ B \\ C \end{array} & \begin{bmatrix} 1/2 & 1/3 & 1/4 \\ 1/3 & 1/3 & 1/4 \\ 1/6 & 1/3 & 1/2 \end{bmatrix} \end{array} = E$$

Solve $(I - E)P = 0$

so solve

$$\begin{bmatrix} 1/2 & -1/3 & -1/4 \\ -1/3 & +2/3 & -1/4 \\ -1/6 & -1/3 & +1/2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = 0$$

$E_q\ 1 - E_q\ 2$ $\qquad \frac{5}{6} P_1 - P_2 = 0$

$P_1 = 6 \quad P_2 = 5$

Plug into $E_q\ 3$

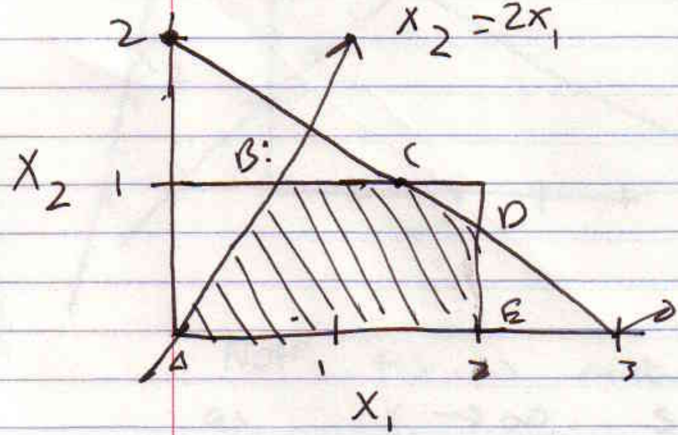
$-\frac{1}{6} \cdot 6 - \frac{1}{3} \cdot 5 + \frac{1}{2} P_3 = 0$

$-\frac{16}{6} + \frac{1}{2} P_3 = 0$ $\qquad$ multiply all by 20

$P_3 = \frac{16}{3} = 8/1.5$!

$$\vec{p} = \begin{bmatrix} 120 \\ 100 \\ 160/1.5 \end{bmatrix}$$

1. $0 \le x_1 \le 2$ $\quad 0 \le x_2 \le 1$



$2x_1 + 3x_2 = 6 \rightarrow$ (note intercepts

Extrem points: A:(0, 0) $B(\frac{1}{2}, 1)$ C: $(\frac{3}{2}, 1)$

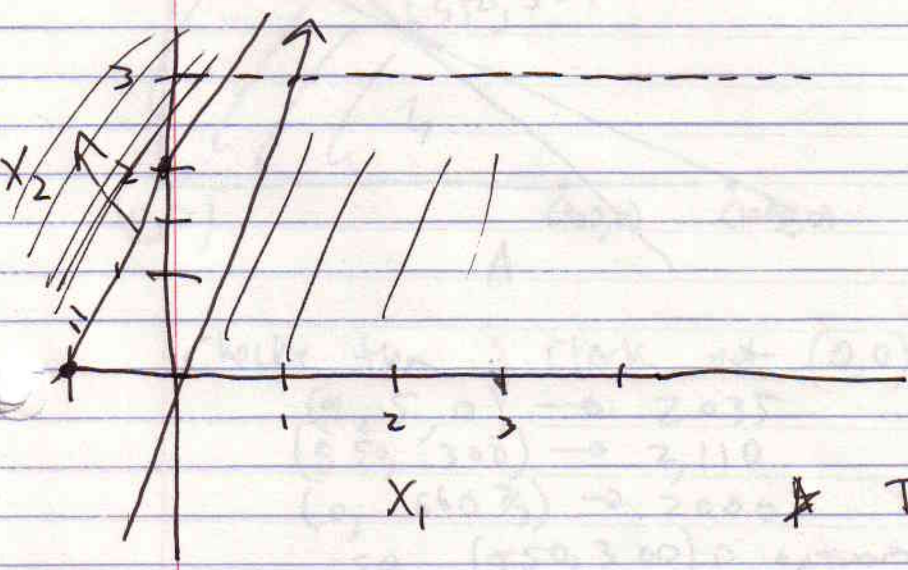
~~D~~ $(2, \frac{2}{3})$ E $(2, 0)$

Values A: 0 B: $\frac{7}{2}$ C: $\frac{13}{2}$ D: $\frac{22}{3}$ E: 6

$\frac{22}{3} = \boxed{7\frac{1}{2}}$ is largest, so D: $\boxed{(2, \frac{2}{3})}$

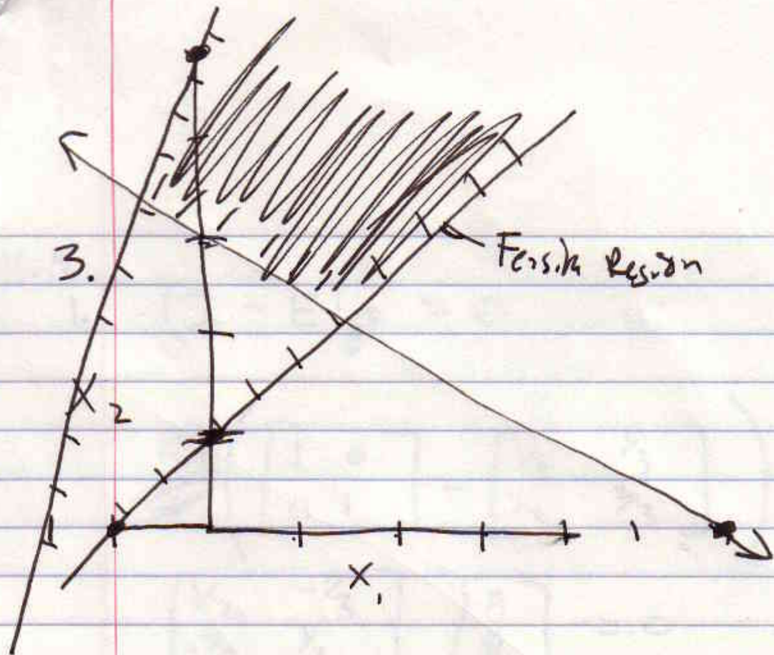
2. $x_1 \ge 0$

$0 \le x_2 \le 3$



* The Feasible region is empty.

3.



Note: this is unbounded, and z goes to near infinity as $X_1 \to \infty$. So, there is no solution.

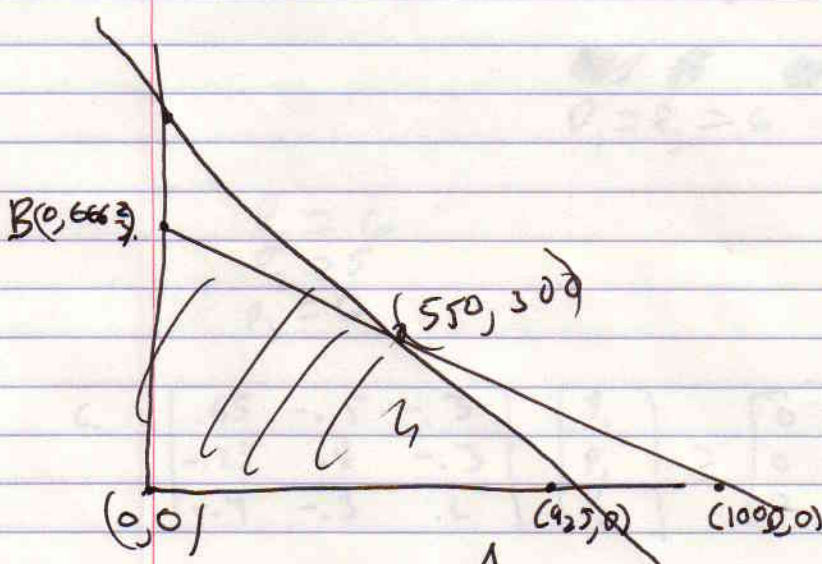
6. Max: Profits.

$2.2A + 3B$

Subject to:

$40A + 50B \leq 37{,}000$

$2A + 3B \leq 2000$



Checker then clerk not (0,0)

$(925, 0) \to 2035$

$(550, 300) \to 2{,}110$

$(0, 666\frac{2}{3}) \to 2000$

so $(550, 300)$ is optimum

~~B~~7. ~~Next~~ Now, check them again:

$(0,0) \to 0$

$(0, 666\frac{2}{3}) \to 2000$

$(550, 300) \to 2{,}275$

$(925, 0) \to 2312.5$

Now, $(925,0)$ is optimum.