Exercise Set 3.2

2.) a) $P_1(3,4)$ $P_2(5,7)$ $d=?$

$d = \|\overrightarrow{P_1P_2}\| = \sqrt{(5-3)^2+(7-4)^2} = \sqrt{4+9} = \sqrt{13}$

b) $P_1(-3,6)$ $P_2(-1,-4)$ $d=?$

$d = \|\overrightarrow{P_1P_2}\| = \sqrt{(-1+3)^2+(-4-6)^2} = \sqrt{4+100} = \sqrt{104} = 2\sqrt{26}$

c) $P_1(7,-5,1)$ $P_2(-7,-2,-1)$

$d = \|\overrightarrow{P_1P_2}\| = \sqrt{(-7-7)^2+(-2+5)^2+(-1-1)^2} = \sqrt{196+9+4} = \sqrt{209}$

d) $P_1(3,3,3)$, $P_2(6,0,3)$

$d = \|\overrightarrow{P_1P_2}\| = \sqrt{(6-3)^2+(0-3)^2+(3-3)^2} = \sqrt{9+9} = 3\sqrt{2}$

4. $\vec{v} = (-1,2,5)$ $\|k\vec{v}\| = 4$ $\|k\vec{v}\| = \sqrt{(k^2+4k^2+25k^2)} = |k|\sqrt{30}$

$|k|\sqrt{30} = 4$ or $k = \pm 4/\sqrt{30}$

6. a) $\frac{1}{\|V\|}\cdot V$ - unitvector $\|\vec{u}\| = 1$

$\frac{1}{\sqrt{v_1^2+v_2^2+v_3^2}}\cdot(v_1,v_2,v_3) = \left(\frac{v_1}{\sqrt{v_1^2+v_2^2+v_3^2}}, \frac{v_2}{v_1^2+v_2^2+v_3^2}, \frac{v_3}{v_1^2+v_2^2+v_3^2}\right) = \vec{u}$ $\|\vec{u}\| = \sqrt{\frac{v_1^2}{v_1^2+v_2^2+v_3^2}+\frac{v_2^2}{v_1^2+v_2^2+v_3^2}+\frac{v_3^2}{v_1^2+v_2^2+v_3^2}}$

$= \sqrt{\frac{v_1^2+v_2^2+v_3^2}{v_1^2+v_2^2+v_3^2}} = \underline{\underline{1}}$ = unit vector

b) From part a) we know that the norm of $\vec{v}/\|\vec{v}\| = 1$

If $\vec{v} = (3,4)$, then $\|\vec{v}\| = 5$. Hence $\vec{u} = \vec{v}/\|\vec{v}\| = \left(\frac{3}{5}, \frac{4}{5}\right)$

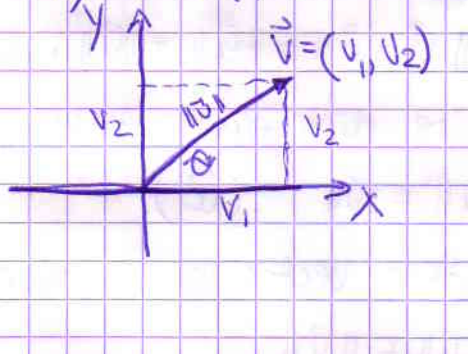
has norm 1 and has the same direction as $\vec{v}$.

c) $-\vec{v}$ has the same length as $\vec{v}$, but is oppositely directed.

$\vec{v} = (-2,3,-6)$ $-\vec{v} = (2,-3,6)$, $\|\vec{v}\| = \sqrt{4+9+36} = 7$

$\vec{u} = \vec{v}/\|\vec{v}\| = \left(\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right)$

7. a) $\vec{v} = (v_1, v_2)$, Show that $v_1 = \|\vec{v}\|\cos\theta$, $v_2 = \|\vec{v}\|\sin\theta$



$\cos\theta = \frac{v_1}{\|\vec{v}\|}$ $v_1 = \|\vec{v}\|\cos\theta$

$\sin\theta = \frac{v_2}{\|\vec{v}\|}$ $v_2 = \|\vec{v}\|\sin\theta$

b) $4\vec{u} = 4(\|\vec{u}\|\cos 30°, \|\vec{u}\|\sin 30°) = 4(3(\sqrt{3}/2), 3(1/2)) = (6\sqrt{3},$

$5\vec{v} = 5(\|\vec{v}\|\cos(135°), \|\vec{v}\|\sin(135°)) = 5(2(-1/\sqrt{2}), 2(1/\sqrt{2})) = (-10/\sqrt{2}, 10/$

$= (-5\sqrt{2}, 5\sqrt{2})$

Thus $4\vec{u} - 5\vec{v} = (6\sqrt{3} + 5\sqrt{2}, 6 - 5\sqrt{2})$

8. $\vec{p}_0 = (x_0, y_0, z_0)$ $\vec{p} = (x, y, z)$

$\|p - p_0\| = 1$

Note that $\|p - p_0\| = 1$ if and only if $\|\vec{p} - \vec{p}_0\|^2 = 1$, Thus

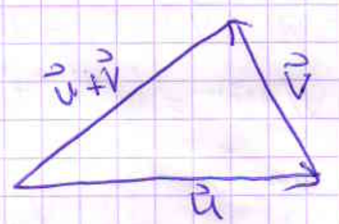
$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = 1$

Points (x, y, z) that satisfy these equations are just the points on the sphere or radius 1, w/ center (x_0, y_0, z_0).

9. $\|\vec{u} + \vec{v}\| \le \|\vec{u}\| + \|\vec{v}\|$

First, suppose that $\vec{u}$ and $\vec{v}$ are neither similarly nor oppositely directed and that neither is the 0 vector.

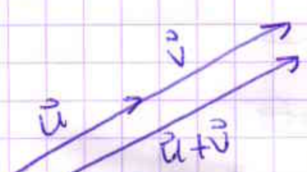
If we place the initial point of $\vec{v}$ at the terminal point of $\vec{u}$, then vectors $\vec{u}, \vec{v}$ and $\vec{u}+\vec{v}$ form a triangle as shown ~~[illegible]~~ below



Since the length of one side of a triangle, say $\|\vec{u}+\vec{v}\|$, cannot exceed the sum of the lengths of the other two sides, then $\|\vec{u}+\vec{v}\| < \|\vec{u}\| + \|\vec{v}\|$.

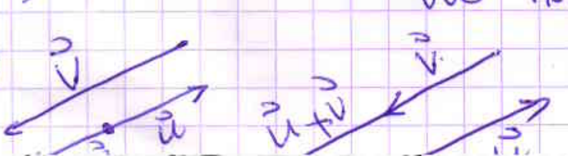
Now suppose that $\vec{u}$ and $\vec{v}$ have the same direction.

i)



$\to$ i) we see that $\|\vec{u}+\vec{v}\| = \|\vec{u}\| + \|\vec{v}\|$. If $\vec{u}$ and $\vec{v}$ have opposite directions, then we have $\|\vec{u}+\vec{v}\| < \|u\| + \|v\|$ (ii)

ii)



Finally, if either vector is 0, then $\|\vec{u}+\vec{v}\| = \|\vec{u}\| + \|\vec{v}\|$

3.3

4) a) $\vec{u} = (6,2) \quad \vec{a} = (3,-9)$

$\text{proj}_{\vec{a}} \vec{u} = \dfrac{\vec{u}\cdot\vec{a}}{\|\vec{a}\|^2} \cdot \vec{a} =$ since $\vec{u}\cdot\vec{a} = 0 \quad \text{proj}_{\vec{a}} \vec{u} = (0,0)$

b) $\vec{u} = (-1,-2) \quad \vec{a} = (-2,3)$

$\text{proj}_{\vec{a}} \vec{u} = \dfrac{(-1)(-2)+(-2)(3)}{13}(-2,3) = \dfrac{-4}{13}(-2,3) = \left(\dfrac{8}{13}, \dfrac{-12}{13}\right)$

c) $\vec{u} = (3,1,-7) \quad \vec{a} = (1,0,5)$

$\text{proj}_{\vec{a}} \vec{u} = \dfrac{(3)(1)+(1)(0)+(-7)(5)}{26}(1,0,5) = \dfrac{-16}{13}(1,0,5) = \left(\dfrac{-16}{13}, 0, \dfrac{-80}{13}\right)$

d) $\vec{u} = (1,0,0) \quad \vec{a} = (4,3,8)$

$\text{proj}_{\vec{a}} \vec{u} = \dfrac{(1)(4)+(0)(3)+(0)(8)}{(\sqrt{89})^2}(4,3,8) = \dfrac{4}{89}(4,3,8) = \left(\dfrac{16}{89}, \dfrac{12}{89}, \dfrac{32}{89}\right)$

5. a) $\vec{u} - \text{proj}_{\vec{a}}\vec{u} = (6,2)-(0,0) = (6,2)$

b) $\vec{u} - \text{proj}_{\vec{a}}\vec{u} = (-1,-2) - \left(\dfrac{8}{13}, -\dfrac{12}{13}\right) = \left(-\dfrac{21}{13}, -\dfrac{14}{13}\right)$

c) $\vec{u} - \text{proj}_{\vec{a}}\vec{u} = (3,1,7) - \left(-\dfrac{16}{3}, 0, -\dfrac{80}{3}\right) = \left(55/13, 1, -11/13\right)$

d) $\vec{u} - \text{proj}_{\vec{a}}\vec{u} = (1,0,0) - \left(16/89, 12/89, 32/89\right) = \left(73/89, -12/89, -32/89\right)$

9. $\vec{u} = (3,4)$, $\vec{v} = (5,-1)$ $\quad \vec{w} = (7,1)$

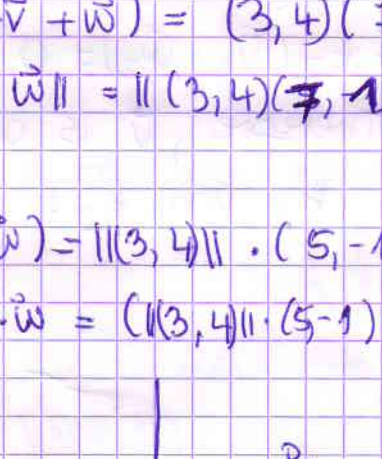
a) $\vec{u} \cdot (7\vec{v} + \vec{w}) = (3,4)(7(5,-1)+(7,1)) = (3,4)\cdot(42,-6) = 102$

b) $\|(\vec{u}\cdot\vec{w})\vec{w}\| = \|(3,4)(7,1)(7,1)\| = \|25(7,1)\| = \|175,25\| =$
$\sqrt{175^2+25^2} = 125\sqrt{2}$

c) $\|\vec{u}\| \cdot (\vec{v}\cdot\vec{w}) = \|(3,4)\| \cdot (5,-1)(7,1) = \sqrt{25}\,(35-1) = 5\cdot(34) = 170$

d) $(\|\vec{u}\|\cdot\vec{v})\cdot\vec{w} = (\|(3,4)\|\cdot(5,-1))\cdot(7,1) = (25,-5)(7,1) = 170$

11. $P_1(0,-1)$
$P_2(1,-2)$
$P_3(4,1)$



$\vec{P_1P_2} = (1,-1)$
$\vec{P_2P_3} = (3,3)$
$\vec{P_1P_3} = (4,2)$

$\cos\theta_1 = \dfrac{\vec{P_1P_2}\cdot\vec{P_1P_3}}{\|P_1P_2\|\|P_2P_3\|} = \dfrac{\sqrt{10}}{10}$

$\cos\theta_2 = \dfrac{\vec{P_2P_1}\cdot\vec{P_2P_3}}{\|\vec{P_2P_1}\|\|\vec{P_2P_3}\|} \quad 0$

$\cos\theta_3 = \dfrac{\vec{P_3P_1}\cdot\vec{P_3P_2}}{\|\vec{P_3P_1}\|\|\vec{P_3P_2}\|} = \dfrac{3\sqrt{10}}{10}$

13.) $\vec{u} = (1,0,1)$ $\vec{v} = (0,1,1)$

Let $\vec{w} = (x, y, z)$ be orthogonal to both $\vec{u}$ and $\vec{v}$. Then $\vec{v} \cdot \vec{w} = 0$ implies $y + z = 0$, and $\vec{u} \cdot \vec{w} = 0$ implies $x + z = 0$.

$\vec{w} = (x, x, -x)$. To transfer $\vec{w}$ into a unit vector, we divide each component by $\|\vec{w}\| = \sqrt{3x^2} = \pm x\sqrt{3}$

Thuse $\vec{w} = (1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3})$ or $(-1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$

15.)

b) $D = \dfrac{|4(2) + (1)(-5) - 2|}{\sqrt{4^2 + 1^2}} = \dfrac{1}{\sqrt{17}}$

a) $D = \dfrac{|(-3)(4) + 3(1) + 4|}{\sqrt{4^2 + 3^2}} = \dfrac{5}{5} = 1$

c) $D = \dfrac{|3(1) + 1(8) - 5|}{\sqrt{10}} = \dfrac{6}{\sqrt{10}}$

16.) $\|\vec{u} + \vec{v}\|^2 + \|\vec{u} - \vec{v}\|^2 = 2\|\vec{u}$

From the definition of the norm, we have

$\|\vec{u} + \vec{v}\|^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v})$. Using Thm. 3.3.2. we get

$\|\vec{u} + \vec{v}\|^2 = (\vec{u} \cdot \vec{u}) + (\vec{u} \cdot \vec{v}) + (\vec{v} \cdot \vec{u}) + (\vec{v} \cdot \vec{v})$ or

$+\Big[$ $\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + 2(\vec{u} \cdot \vec{v}) + \|\vec{v}\|^2$

Similarly $\|\vec{u} - \vec{v}\|^2 = \|\vec{u}\|^2 - 2(\vec{u} \cdot \vec{v}) + \|\vec{v}\|^2$

If we add the last 2 equations we get: $2\|\vec{u}\|^2 + 2\|\vec{v}\|^2$

22) $\vec{v} \cdot (k_1\vec{w}_1 + k_2\vec{w}_2) = k_1(\vec{v} \cdot \vec{w}_1) + k_2(\vec{v} \cdot \vec{w}_2) = 0$ because, by hypothesis, $\vec{v} \cdot \vec{w}_1 = \vec{v} \cdot \vec{w}_2 = 0$. Therefore, $\vec{v}$ is orthogonal to $k_1\vec{w}_1 + k_2\vec{w}_2$ for any scalars k_1 and k_2.