

$$5. \begin{bmatrix} \cancel{1} & \cancel{1} \\ \cancel{1} & \cancel{3} \\ \cancel{1} & \cancel{7} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 3 \\ 1 & -1 \end{bmatrix} b. \begin{bmatrix} 7 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} c. \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} d. \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$6. = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad b. \begin{bmatrix} 3 \\ 13 \end{bmatrix}$$

$$c. \begin{bmatrix} -2x_1 + x_2 + 4x_3 \\ 3x_1 + 5x_2 + 7x_3 \\ 6x_1 + x_3 \end{bmatrix} \quad d. \begin{bmatrix} -x_1 + x_2 \\ 2x_1 + 4x_2 \\ 7x_1 + 6x_2 \end{bmatrix}$$

$$12. = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 3 \\ -7 \end{bmatrix}$$

$$a. \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -7 \end{bmatrix} = \begin{bmatrix} \frac{3\sqrt{3}}{2} + 2 \\ \frac{3}{2} - 2\sqrt{3} \end{bmatrix}$$

$$b. \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -7 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} - 2\sqrt{3} \\ -\frac{3\sqrt{3}}{2} - 2 \end{bmatrix}$$

$$c. \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -7 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ -\sqrt{3} \end{bmatrix}$$

$$d. \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -7 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$13. \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ \frac{\sqrt{3}}{2} - \frac{1}{2} \\ \frac{1}{2} + \sqrt{3} \end{bmatrix}$$

$$b. \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\sqrt{3} \\ 1 \\ \sqrt{3} \end{bmatrix}$$

$$c. \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$$

$$16. \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

rot $y=x$ rot 90

$$b \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$c. \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix}$$

17.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$b. \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix} = \begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$$

$$c. \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix}$$

rows; we prove they add

~~rot 60~~

$$(\text{rot } 60) \cdot (\text{rot } 105) \cdot (\text{rot } 15) = (\text{rot } 60) \cdot (\text{rot } 120) = (\text{rot } 180)$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$20. T_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$T_1 \circ T_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$T_2 \circ T_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

yes

Same

$$b. T_1 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 \\ \sin \theta_2 & \cos \theta_2 \end{bmatrix}$$

proved

$$T_2 \circ T_1 = \text{rot}(\theta_1 + \theta_2) = \text{rot}(\theta_2 + \theta_1) = T_1 \circ T_2 \quad \text{yes}$$

c.

$$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$T_1 \circ T_2 = \begin{bmatrix} \cos \theta & -\sin \theta \\ 0 & 0 \end{bmatrix}$$

$$T_2 \circ T_1 = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \end{bmatrix}$$

no

$$21. T_1 = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} = k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So any matrix

$$T_2 = B$$

$$T_1 T_2 = k I B = k B I = \frac{B}{k} (Ik) = T_2 T_1$$

including the rotation matrix

yes

$$b. T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & -\sin \theta_1 \\ 0 & \sin \theta_1 & \cos \theta_1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 \\ \sin \theta_2 & \cos \theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_1 T_2 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 \\ \cos \theta_1 \cos \theta_2 & \cos \theta_1 \sin \theta_2 & -\sin \theta_1 \\ \sin \theta_1 \cos \theta_2 & \sin \theta_1 \sin \theta_2 & \cos \theta_1 \end{bmatrix}$$

$$T_2 T_1 = \begin{bmatrix} \cos \theta_2 & -\cos \theta_1 \sin \theta_2 & \sin \theta_1 \sin \theta_2 \\ \sin \theta_2 & \cos \theta_1 \cos \theta_2 & -\sin \theta_1 \cos \theta_2 \\ 0 & \sin \theta_1 & \cos \theta_1 \end{bmatrix}$$

So they are not equal