Section 5.2

7. a) $(2,2,2) = k_1 u + k_2 v$
$(2,2,2) = k_1(0,-2,2) + k_2(1,3,-1)$
$(2,2,2) = (k_2, -2k_1+3k_2, 2k_1-k_2)$

$k_2 = 2$
$-2k_1 + 3k_2 = 2$
$2k_1 - k_2 = 2$

$k_1 = 2 \quad k_2 = 2$
$(2,2,2) = 2u + 2v$
linear combination

b) $(3,1,5) = k_1 u + k_2 v$
$(3,1,5) = k_1(0,-2,2) + k_2(1,3,-1)$
$(3,1,5) = (k_2, -2k_1+3k_2, 2k_1-k_2)$

$k_2 = 3$
$-2k_1 + 3k_2 = 1$
$2k_1 - k_2 = 5$

$k_1 = 4 \quad k_2 = 3$
$(3,1,5) = 4u + 3v$
linear comb.

c) $(0,4,5) = k_1 u + k_2 v$
$(0,4,5) = k_1(0,-2,2) + k_2(1,3,-1)$
$(0,4,5) = (k_2, -2k_1+3k_2, 2k_1-k_2)$

$k_2 = 0$
$-2k_1 + 3k_2 = 4$
$2k_1 - k_2 = 5$

Inconsistent - not a linear comb.

d) $(0,0,0) = k_1 u + k_2 v$
$= (k_2, -2k_1+3k_2, 2k_1-k_2)$

$k_2 = 0$
$-2k_1 + 3k_2 = 0$
$2k_1 - k_2 = 0$

$k_1 = 0 \quad k_2 = 0$
$(0,0,0) = 0u + 0v$
linear comb

8. b) $a(2,1,4) + b(1,-1,3) + c(3,2,5) = (6,11,6)$
$2a + b + 3c = 6$
$a - b + 2c = 11$
$4a + 3b + 5c = 6$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 3 & 6 \\ 1 & -1 & 2 & 11 \\ 4 & 3 & 5 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$a = 4, \quad b = -5$
$c = 1 \Rightarrow$ so $(6,11,6)$ is a linear comb. of $u, v,$ and w.

c) $a(2,1,4) + b(1,-1,3) + c(3,2,5) = (0,0,0)$
$2a + b + 3c = 0$
$a - b + 2c = 0$
$4a + 3b + 5c = 0$

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 1 & -1 & 2 & 0 \\ 4 & 3 & 5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$a = 0 \quad b = 0 \quad c = 0$, so $(0,0,0)$ is a linear comb of u, v, w.

Section 5.3

2. a) Neither vector is a linear multiple of the other, so by Thm 5.3.2 they are linearly indep.

b) $-3k_1 + 5k_2 + k_3 = 0$
$-k_2 + k_3 = 0$
$4k_1 + 2k_2 + 3k_3 = 0$

$k_2 = k_3 \Rightarrow k_1 = 2k_2 = 2k_3$, thus $k_1 = k_2 = k_3 = 0 \Rightarrow$ only solution, $\Rightarrow$ linearly independent

c) Same as a)

d) By Thm 5.3.3 vectors are linearly independent, because any 4 vectors in $\mathbb{R}^2$ are linearly indep. $(4 > 3)$

7. a) $a(0,3,1,-1)+b(6,0,5,1)+c(4,-7,1,3)=(0,0,0,0)$

$6b+4c=0$
$3a-7c=0$
$a+5b+c=0$
$-a+b+3c=0$

$$\Rightarrow \begin{vmatrix} 0 & 6 & 4 & 0 \\ 3 & 0 & -7 & 0 \\ 1 & 5 & 1 & 0 \\ -1 & 1 & 3 & 0 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 0 & -7/3 & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0$$

det $=0$, which means that the system has trivial solution, and thus the 3 vectors are linearly independent.

or, ~~if~~ $c=3$, $b=-2$ $a=7$ $\rightarrow$ $7v_1-2v_2+3v_3=0$

b) $v_1 = bv_2 + cv_3$

$(0,3,1,-1) = b(6,0,5,1)+c(4,-7,1,3)$

$6b+4c=0$
$-7c=3$
$5b+c=1$
$b+3c=-1$

$c=-3/7$
$b=2/7$

$$\underline{v_1 = \tfrac{2}{7}v_2 - \tfrac{3}{7}v_3}$$

$v_2 = av_1 + cv_3$

$(6,0,5,1) = a(0,3,1,-1)+c(4,-7,1,3)$

$4c=6$
$3a-7c=0$
$a+c=5$
$-a+3c=1$

$c=3/2$
$a=7/2$

$$\underline{v_2 = \tfrac{7}{2}v_1 + \tfrac{3}{2}v_3}$$

$v_3 = av_1 + bv_2$

$(4,-7,1,3) = a(0,3,1,-1)+b(6,0,5,1)$

$6b=4$
$3a=-7$
$a+5b=1$
$-a+b=3$

$b=2/3$
$a=-7/3$

$$\underline{v_3 = -\tfrac{7}{3}v_1 + \tfrac{2}{3}v_2}$$

9. $S=\{v_1,v_2,v_3\}$ lin. independent if $k_1v_1+k_2v_2+k_3v_3=0$, has solution $k_1=k_2=k_3=0$

According to Thm 5.3.1 for n vectors to be linearly independent, no vectors can be written as a linear combination of another. Since any of v_1,v_2,v_3 cannot be written as a linear combination of one another, then a set any ~~~~ two vectors $\Rightarrow \{v_1,v_2\}, \{v_1,v_3\}$ or $\{v_2,v_3\}$ are also linearly independent.

!

Section 5.4

2.a) Vectors are not multiple of each other. → set is a basis

b) same as a)

c) This set is not a basis, b/c (0,0) is a scalar multiple of (1,3)

d) $\det\begin{pmatrix} 3 & 9 \\ 4 & -12 \end{pmatrix} = 0$, therefore this set is not a basis

3. b) $\det A = \begin{vmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{vmatrix} = 6 \neq 0$

Since $\det A \neq 0$, vectors are linearly independent → set is a basis

c) $\det A = \begin{vmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 8 & 9 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 0 & 0 \end{vmatrix} = 0$

Since $\det A = 0$, vectors are lin. dependent, thus the set is not a basis.