7.2.

9.) $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$ $\det(\lambda I - A) = \begin{vmatrix} \lambda-1 & 0 \\ 6 & -1-\lambda \end{vmatrix} = (\lambda-1)(-1-\lambda)$ $(\lambda-1)(\lambda+1)=0$

$\underline{\lambda_1 = 1 \quad \lambda_2 = -1}$

$\lambda_1 = 1$ $\begin{bmatrix} 0 & 0 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $6x_1 - 2x_2 = 0$

$x_1 = \frac{1}{3}x_2$ let $x_2 = s$ $x = \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} s$ $p_1 \begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$

$\lambda_2 = -1$ $\begin{bmatrix} -2 & 0 \\ -6 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $-2x_1 = 0$ $-6x_1 = 0$ $x = \begin{bmatrix} 0 \\ 1 \end{bmatrix} s$ $p_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 1/3 & 0 \\ 1 & 1 \end{bmatrix}$ $P^{-1}AP = \begin{bmatrix} 1 & 0 \\ -1 & 1/3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 1/3 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 \\ 0 & -1/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

11.) $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $\det(\lambda I - A) = \begin{vmatrix} \lambda-2 & 0 & 2 \\ 0 & \lambda-3 & 0 \\ 0 & 0 & \lambda-3 \end{vmatrix} = (\lambda-2)(\lambda-3)^2$

$\underline{\lambda = 2, 3}$

$\lambda = 2$ $(\lambda I - A) \Rightarrow \begin{bmatrix} 0 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $2x_3 = 0$ $-x_2 = 0$ $-x_3 = 0$ $x = \begin{bmatrix} s \\ 0 \\ 0 \end{bmatrix}$ $p = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 3$ $(\lambda I - A) \Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $x_1 + 2x_3 = 0$ $x_1 = -2x_3$ $x = \begin{bmatrix} -2s \\ t \\ s \end{bmatrix}$ $p = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ $p = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$P = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$

$P^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $P^{-1}AP = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

14.) $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$ $\det(\lambda I - A) = \begin{bmatrix} \lambda-5 & 0 & 0 \\ -1 & \lambda-5 & 0 \\ 0 & -1 & \lambda-5 \end{bmatrix} = (\lambda-5)^3$ $\underline{\lambda = 5}$

There is only one destinct eigenvalue, thus A is not diagonalizable.

19.) $\det(\lambda I - A) = \begin{vmatrix} \lambda+1 & -7 & 1 \\ 0 & \lambda-1 & 0 \\ 0 & -15 & \lambda+2 \end{vmatrix} = (\lambda+1)(\lambda-1)(\lambda+2)$ $\lambda = -2, -1, 1$

$\lambda = -2$ $(\lambda I - A) \Rightarrow \begin{bmatrix} -1 & -7 & 1 \\ 0 & -3 & 0 \\ 0 & -15 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $-x_1 - 7x_2 + x_3 = 0$, $-3x_2 = 0$, $-15x_2 = 0$

$x = \begin{bmatrix} s \\ 0 \\ s \end{bmatrix}$ $P_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\lambda = -1$ $\begin{bmatrix} 0 & -7 & 1 \\ 0 & -2 & 0 \\ 0 & -15 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $-7x_2 + x_3 = 0$, $-2x_2 = 0$, $-15x_2 + x_3 = 0$ $x = \begin{bmatrix} s \\ 0 \\ 0 \end{bmatrix}$ $P_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 1$ $\begin{bmatrix} 2 & -7 & 1 \\ 0 & 0 & 0 \\ 0 & -15 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $2x_1 - 7x_2 + x_3 = 0$, $-15x_2 + 3x_3 = 0$ $x = \begin{bmatrix} 1/5\, s \\ 1/5\, s \\ s \end{bmatrix}$ $P_3 = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}$ $P^{-1} = \begin{bmatrix} 0 & -5 & 1 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -5 & 1 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A^{11} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}\begin{bmatrix} -2^{11} & 0 & 0 \\ 0 & -1^{11} & 0 \\ 0 & 0 & 1^{11} \end{bmatrix}\begin{bmatrix} 0 & -5 & 1 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 10,237 & -2047 \\ 0 & 1 & 0 \\ 0 & 10245 & -2048 \end{bmatrix}$

Owl Problem

$\begin{bmatrix} j_{k+1} \\ s_{k+1} \\ a_{k+1} \end{bmatrix} = \begin{bmatrix} 0 & 0.125 & 0.26 \\ 0.33 & 0 & 0 \\ 0 & 0.85 & 0.85 \end{bmatrix}\begin{bmatrix} j_k \\ s_k \\ a_k \end{bmatrix}$

$\det\begin{bmatrix} \lambda & -.125 & -.26 \\ -.33 & \lambda & 0 \\ 0 & -.85 & \lambda-.85 \end{bmatrix} = \lambda^3 - 0.85\lambda^2 - 0.04125\lambda - 0.0379 = 0$

$\lambda_1 = 0.9371$ $\lambda_2 > \lambda_1$ $\lambda_3 > \lambda_1$

$x^k = A^k x^0 \Rightarrow \lambda_1^k c_1 x_1 = (0.937)^k c_1 x_1$

$(0.937)^k \to 0$ as $k \to \infty$

Thus the owl population will become extinct.
(It decreases by approximately 6% per year)