Solution Set: 11.4

2) $z = e^{x^2-y^2}$ $(1,-1,1)$

$f_x(x,y) = 2xe^{x^2-y^2} = 2(1)e^0 = 2$

$f_y(x,y) = -2ye^{x^2-y^2} = -2(-1)e^0 = 2$

$z = 1 + 2(x-1) + 2(y+1)$

$= 1 + 2x - 2 + 2y + 2$

$z = 2x + 2y + 1$

4) $z = y\ln x$ $(1,4,0)$

$f_x(x,y) = \frac{y}{x} = 4$

$f_y(x,y) = \ln x = 0$

$z = 4(x-1) - 0(x-4)$

$z = 4x - 4$

10) $f(x,y) = x/y$ $(6,3)$ $f(6,3) = 2$

f is differentiable if f_x and f_y exist and are continuous at $(6,3)$

$f_x = 1/y$ $f_y = -x/y^2$

$f_x(6,3) = 1/3$ $f_y(6,3) = -2/3$

$L(x,y) = 2 + \frac{1}{3}(x-6) - \frac{2}{3}(y-3)$

$= \frac{1}{3}x - \frac{2}{3}y + 2$

12) $f(x,y) = \sin(2x+3y)$ $(-3,2)$ $f(-3,2) = \sin(0) =$

$f_x = 2\cos(2x+3y)$ $f_y = 3\cos(2x+3y)$

$f_x(-3,2) = 2$ $f_y(-3,2) = 3$

differentiable because f_x and f_y exist at $(-3,2)$ and are continuous there.

$L(x) = 2(x+3) + 3(y-2)$

$= 2x + 3y$

13) $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ $(2,1)$

$f(2,1) = \sqrt{20-4-7} = \sqrt{9} = 3$ $(2,1,3)$

$f_x = \dfrac{-2x}{2\sqrt{20-x^2-7y^2}}$ $f_y = \dfrac{-14y}{2\sqrt{20-x^2-7y^2}}$

$f_x(2,1) = \dfrac{-4}{2(3)} = \dfrac{-2}{3}$ $f_y(2,1) = \dfrac{-14}{2(3)} = \dfrac{-7}{3}$

$L(x,y) = 3 - \frac{2}{3}(x-2) - \frac{7}{3}(y-1)$

$= -\frac{2}{3}x - \frac{7}{3}y + \frac{20}{3}$

$L(1.95, 1.08) = 2.846$

4) $f(x,y) = \ln(x-3y)$ $(7,2)$

$f(7,2) = \ln(1) = 0$

$f_x = 1/x-3y$ $f_y = -3/x-3y$

$f_x(7,2) = 1$ $f_y(7,2) = -3$

$L(x,y) = (x-7) - 3(y-2)$

$= x - 3y - 1$

$f(6.9, 2.06) = -0.28$

Solution Set: 11.6

4) $f(x,y) = \sin(x+2y) \quad (4,-2) \quad \theta = 3\pi/4$

$D_u f(x,y) = f_x(x,y)\cos\theta + f_y(x,y)\sin\theta$

$f_x = \cos(x+2y) \qquad f_y = 2\cos(x+2y)$

$f_x(4,-2) = \cos(0) = 1 \qquad f_y(4,-2) = 2$

$D_u f(x,y) = \cos\theta + 2\sin\theta$

$= -\sqrt{2}/2 + 2\sqrt{2}/2$

$D_u f(4,-2) = \sqrt{2}/2$

6) $f(x,y) = xe^{-2y} \quad (5,0) \quad \theta = \pi/2$

$f_x = e^{-2y} = e^0 = 1$

$f_y = -2xe^{-2y} = -10$

$D_u f(x,y) = \cos\theta - 10\sin\theta$

$= 0 - 10$

$= -10$

8) $f(x,y) = y\ln x \quad P(1,-3) \quad u = \langle -4/5, 3/5 \rangle$

$f_x = y/x \qquad f_y = \ln x$

ⓐ $\nabla f = (y/x, \ln x)$

ⓑ $\nabla f(1,-3) = (-3, 0)$

ⓒ $D_u f = \nabla f \cdot \vec{u}$

$(-3,0) \cdot (-4/5, 3/5)$

$= 12/5$

10) $f(x,y,z) = xy + yz^2 + xz^3 \quad P(2,0,3) \quad u = (-2/3$

$f_x = y + z^3 \quad f_y = x + z^2 \quad f_z = 2yz + 3xz^2$

ⓐ $\nabla f = (y+z^3, x+z^2, 2yz + 3xz^2)$

ⓑ $\nabla f(2,0,3) = (27, 11, 54)$

ⓒ $(27, 11, 54) \cdot (-2/3, -1/3, 2/3) = -18 - 11/3 + 36$

$= 43/3$

4) $f(x,y,z) = x/(y+z)$ $(4,1,1)$ $v = \langle 1,2,3\rangle$

$f_x = 1/y+z = 1/2$ $\quad 1^2+2^2+3^2 = \sqrt{14}$ $\quad v = (1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14})$

$f_y = -x/(y+z)^2 = -4/4 = -1$

$f_z = -x/(y+z)^2 = -1$

$\nabla f = (1/2, -1, -1)$

$D_u f = Df \cdot v = (1/2, -1, -1) \cdot (1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14})$

$= 1/2\sqrt{14} - 4/2\sqrt{14} - 6/2\sqrt{14} = -11/2\sqrt{14}$

$D_u f = -11/2\sqrt{14}$

20) $f(x,y) = \ln(x^2+y^2)$ $(1,2)$

$f_x = 2x/(x^2+y^2) = 2/1+4 = 2/5$

$f_y = 2y/(x^2+y^2) = 4/5$

$\nabla f = (2/5, 4/5)$

max. rate of change at $(1,2)$ is $\|\nabla f(1,2)\|$

$|2/5, 4/5| = (2/5)^2 + (4/5)^2$

$4/25 + 16/25 = \sqrt{20/25} = \sqrt{4/5} = 2/\sqrt{5}$ max. rate of change

direction of max. increase $= (2/5, 4/5)$

2) $f(x,y,z) = x^2y^3z^4$ $(1,1,1)$

$f_x = 2xy^3z^4 = 2$

$f_y = 3x^2y^2z^4 = 3$

$f_z = 4x^2y^3z^3 = 4$

$\nabla f(1,1,1) = (2,3,4)$ ← direction of max. increase

$\|(2,3,4)\| = \sqrt{4+9+16} = \sqrt{29}$ ← max. rate of change

11.6 (cont'd)

28) $T(x,y,z) = 200e^{-x^2-3y^2-9z^2}$

ⓐ $\nabla T(x,y,z) = (-400xe^{-x^2-3y^2-9z^2}, -1200ye^{-x^2-3y^2-9z^2}, -3600ze^{-x^2-3y^2-9z^2}$

$\nabla T(2,-1,2) = (-800e^{-43}, 1200e^{-43}, -7200e^{-43})$

direction $\vec{v} = (3,-3,3) - (2,-1,2) = (1,-2,1)$

$\vec{u} = \vec{v}/\|\vec{v}\| = 1/\sqrt{6}\,(1,-2,1)$

$D_{\vec{u}}T(2,-1,2) = \nabla T(2,1,2) \cdot \vec{u}$

$= 1/\sqrt{6}\,[-800e^{-43} - 2400e^{-43}, -7200e^{-43}]$

$= -1/\sqrt{6}\; 10400e^{-43}$

ⓑ temperature increases fastest in direction of the gradient at P

$\nabla T(2,-1,2) = (-800e^{-43}, 1200e^{-43}, -7200e^{-43})$

ⓒ max. rate of increase of r is the magnitude of the gradient at P

$\|\nabla T(2,-1,2)\| = \sqrt{800^2 + 1200^2 + 7200^2} \cdot e^{-43}$

$\approx 7343e^{-43}$