

$$2. \quad A + 2B = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} + 2 \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 2+14 & 0-10 & -1+2 \\ 4+2 & -5-8 & 2-6 \end{bmatrix} = \begin{bmatrix} 16 & -10 & 1 \\ 6 & -13 & -4 \end{bmatrix}$$

The expression $3C - E$ is not defined because $3C$ has 2 columns and $-E$ has only 1 column.

$$CB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 1 \cdot 7 + 2 \cdot 1 & 1(-5) + 2(-4) & 1 \cdot 1 + 2(-3) \\ -2 \cdot 7 + 1 \cdot 1 & -2(-5) + 1(-4) & -2 \cdot 1 + 1(-3) \end{bmatrix} = \begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix}$$

The product EB is not defined because the number of columns of E does not match the number of rows of B .

$$6. \text{ a. } A\mathbf{b}_1 = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 13 \end{bmatrix}, \quad A\mathbf{b}_2 = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 14 \\ -9 \\ 4 \end{bmatrix}$$

$$AB = [A\mathbf{b}_1 \quad A\mathbf{b}_2] = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}$$

$$\text{b. } \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 \cdot 1 - 2 \cdot 2 & 4 \cdot 3 - 2(-1) \\ -3 \cdot 1 + 0 \cdot 2 & -3 \cdot 3 + 0(-1) \\ 3 \cdot 1 + 5 \cdot 2 & 3 \cdot 3 + 5(-1) \end{bmatrix} = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}$$

$$10. AB = \begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}, AC = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$$

18. The first two columns of AB are Ab_1 and Ab_2 . They are equal since b_1 and b_2 are equal.

20. The second column of AB is also all zeros because $Ab_2 = A\mathbf{0} = \mathbf{0}$.

22. If the columns of B are linearly dependent, then there exists a nonzero vector \mathbf{x} such that $B\mathbf{x} = \mathbf{0}$. From this, $A(B\mathbf{x}) = A\mathbf{0}$ and $(AB)\mathbf{x} = \mathbf{0}$ (by associativity). Since \mathbf{x} is nonzero, the columns of AB must be linearly dependent.

26. Write $I_3 = [\mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3]$ and $D = [\mathbf{d}_1 \ \mathbf{d}_2 \ \mathbf{d}_3]$. By definition of AD , the equation $AD = I_3$ is equivalent to the three equations $A\mathbf{d}_1 = \mathbf{e}_1$, $A\mathbf{d}_2 = \mathbf{e}_2$, and $A\mathbf{d}_3 = \mathbf{e}_3$. Each of these equations has at least one solution because the columns of A span \mathbf{R}^3 . (See Theorem 4 in Section 1.4.) Select one solution of each equation and use them for the columns of D . Then $AD = I_3$.

28. Since the inner product $\mathbf{u}^T \mathbf{v}$ is a real number, it equals its transpose. That is,

$\mathbf{u}^T \mathbf{v} = (\mathbf{u}^T \mathbf{v})^T = \mathbf{v}^T (\mathbf{u}^T)^T = \mathbf{v}^T \mathbf{u}$, by Theorem 3(d) regarding the transpose of a product of matrices and by Theorem 3(a). The outer product $\mathbf{u} \mathbf{v}^T$ is an $n \times n$ matrix. By Theorem 3, $(\mathbf{u} \mathbf{v}^T)^T = (\mathbf{v}^T)^T \mathbf{u}^T = \mathbf{v} \mathbf{u}^T$.