Orthogonal Projection In-class Problems

Math 20

November 12, 2004

1. Let

$$\mathbf{y} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \qquad W = \text{Span}\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}.$$

Find vectors $\hat{\mathbf{y}} \in W$ and $\mathbf{z} \in W^{\perp}$ such that $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$.

2. Let

$$\mathbf{y} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \qquad W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}.$$

Find vectors $\hat{\mathbf{y}} \in W$ and $\mathbf{z} \in W^{\perp}$ such that $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$.

3. Let

$$\mathbf{y} = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}, \qquad W = \text{Span}\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}.$$

Find vectors $\hat{\mathbf{y}} \in W$ and $\mathbf{z} \in W^{\perp}$ such that $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$.

4. Let

$$\mathbf{y} = \begin{bmatrix} 2 \\ 4 \\ 1 \\ 3 \end{bmatrix}, \qquad W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

Find vectors $\hat{\mathbf{y}} \in W$ and $\mathbf{z} \in W^{\perp}$ such that $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$.