

## In-Class Problems for Section 6.5

Math 20

November 17, 2004

1. Find a least-squares solution of the system of linear equations

$$\begin{array}{rcl} x_1 & - & 2x_2 = 0 \\ -x_1 & + & x_2 = 1 \\ x_1 & - & x_2 = 1 \end{array}$$

2. Find a least-squares solution of the system of linear equations

$$\begin{array}{rcl} x_1 & + & x_2 + x_3 = 1 \\ & - & x_2 + x_3 = 0 \\ x_1 & + & x_2 = 3 \\ 2x_1 & + & 2x_2 = 4 \end{array}$$

3. Let  $C$  be a matrix such that  $C^T C$  is invertible (this amounts to the columns of  $C$  being linearly independent), and let  $W = \text{Col}(C)$ . Define

$$P = C(C^T C)^{-1} C^T.$$

Show

- (a)  $P^2 = P$
- (b) If  $\mathbf{v} \in W$ , then  $P\mathbf{v} = \mathbf{v}$ .
- (c) If  $\mathbf{v} \in W^\perp$ , then  $P\mathbf{v} = \mathbf{0}$ .