

Hungarian method for assignment problem

Step 1. Subtract the entries of each row by the row minimum.

Step 2. Subtract the entries of each column by the column minimum.

Step 3. Make an assignment to the zero entries in the resulting matrix.

$$\mathbf{A} = \begin{bmatrix} M & 17 & 10 & 15 & 17 \\ 18 & M & 6 & 10 & 20 \\ 12 & 5 & M & 14 & 19 \\ 12 & 11 & 15 & M & 7 \\ 16 & 21 & 18 & 6 & M \end{bmatrix} \begin{matrix} -10 \\ -6 \\ -5 \\ -7 \\ -6 \end{matrix} \longrightarrow \begin{bmatrix} M & 7 & 0 & 5 & 7 \\ 12 & M & 0 & 4 & 14 \\ 7 & 0 & M & 9 & 14 \\ 5 & 4 & 8 & M & 0 \\ 10 & 15 & 12 & 0 & M \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} M & 7 & 0 & 5 & 7 \\ 7 & M & 0 & 4 & 14 \\ 2 & 0 & M & 9 & 14 \\ 0 & 4 & 8 & M & 0 \\ 5 & 15 & 12 & 0 & M \end{bmatrix} \xRightarrow{\text{Assignment}} \begin{bmatrix} & & \otimes & & \\ & & 0 & & \\ & & & \otimes & \\ \otimes & & & & \\ & & & & \otimes & 0 \end{bmatrix}$$

If there are not enough zeros for making a complete assignment, use Step 4 to generate more zeros for assignment.

Step 4

- Mark the unassigned rows.
- Mark columns which has zeros in marked rows.
- Mark rows which has assignments in marked columns. Go to second item.
- Draw a line to cover each unmarked row and each marked column.
- Let m be the minimum of the uncovered entries. Subtract m from each marked row and add m to each marked column.

Step 5 Go to Step 3.