

Name: \_\_\_\_\_ ID#: \_\_\_\_\_

# Midterm Exam I

Math 20  
Introduction to Linear Algebra  
and Multivariable Calculus

8 March 2004

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

**This is a non-calculator exam.**

*Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.*

*—Handbook for Students*

**In all situations where row operations are performed on a matrix, label each operation to receive partial credit in case of arithmetic mistakes.**

Problem Number	Possible Points	Points Earned
1	25	
2	20	
3	20	
4	10	
5	12	
6	13	
Total	100	

**1****1**

1. (25 Points) Let

$$A = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}; \quad B = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}.$$

Find the following:

(i)  $3A + 2B$

(ii)  $AB$

**1**

**1**

(iii)  $A^2B^T$

**2****2**

**2.** (20 Points) Find the parametric form to the general solution of the system of linear equations

$$\begin{aligned}2x_1 + 4x_2 + x_4 &= 2; \\x_1 + 2x_2 + x_3 + 2x_4 &= 4; \\3x_1 + 6x_2 + 2x_3 + x_4 &= 2.\end{aligned}$$

**3**

**3**

3. (20 Points) Determine whether set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ -14 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \\ 2 \end{bmatrix} \right\}$$

is linearly dependent.

**4**

**4**

4. (10 Points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation that rotates counterclockwise by  $30^\circ$  then reflects in the  $x_2$ -axis. What is the matrix associated to this linear transformation?

5. (12 Points) Label the following statements as true or false. Justify your answers. (If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true *in every case*.)

\_\_\_\_\_ (a) Any system of  $n$  linear equations in  $n$  variables has at most  $n$  solutions.

\_\_\_\_\_ (b) If the system  $A\mathbf{x} = \mathbf{b}$  has more than one solution, then so does the system  $A\mathbf{x} = \mathbf{0}$ .

\_\_\_\_\_ (c) If  $A$ ,  $B$ , and  $C$  are matrices,  $AB = C$  and  $C$  has 3 columns, then  $A$  has three columns.

\_\_\_\_\_ (d) If  $A$  is square and has a pivot position in every row, then there exists a matrix  $B$  such that  $AB = I$ .

6

6

6. (13 Points) Let  $A$  be an  $n \times n$  matrix that is not invertible. Show there exists an  $n \times n$  matrix  $B$  such that  $AB$  is the  $n \times n$  matrix of zeroes. (Of course you could pick  $B$  to be the  $n \times n$  matrix of zeroes, so let's not count that one!)

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