Name: ______________________ ID#: ______________

# Midterm Examination II

Math 20
Introduction to Linear Algebra
and Multivariable Calculus

21 April 2004

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

**This is a non-calculator exam.**

> *Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.*
> *—Handbook for Students*

**In all situations where row operations are performed on a matrix, label each operation to receive partial credit in case of arithmetic mistakes.**

| Problem Number | Possible Points | Points Earned |
|---|---|---|
| 1 | 15 | |
| 2 | 15 | |
| 3 | 15 | |
| 4 | 15 | |
| 5 | 15 | |
| 6 | 9 | |
| 7 | 16 | |
| Total | 100 | |

**1.** (15 Points) Let $W$ be the subspace spanned by

$$\left\{ \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ 3 \end{bmatrix} \right\}$$

Find a basis for $W$ and the dimension of $W$.



_____ / 15

**2.** (15 Points) Find all numbers $c$ such that

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & c \\ 0 & c & -15 \end{bmatrix}$$

is not invertible.



______ / 15

**3.** (15 Points) Let

$$A = \begin{bmatrix} \frac{11}{10} & \frac{2}{5} \\ \frac{3}{5} & \frac{9}{10} \end{bmatrix}.$$

Consider the discrete dynamical system defined by the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$. Is $\mathbf{0}$ an attractor? repellor? saddle point? Why?



______ / 15

**4.** (15 Points) Let $A$ be the matrix $\begin{bmatrix} -2 & -5 & 2 \\ 4 & 7 & -2 \\ -3 & -3 & 5 \end{bmatrix}$. 5 is an eigenvalue of $A$. Give the eigenvector of unit length corresponding to this eigenvalue.



_____ / 15

**5.** (15 Points) Let

$$\mathbf{w_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}; \quad \mathbf{w_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}; \quad \mathbf{w_3} = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}; \quad \mathbf{v} = \begin{bmatrix} 2 \\ 4 \\ 1 \\ 3 \end{bmatrix}.$$

Let $W = \text{Span}\{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$. Compute $\hat{\mathbf{v}} = \text{proj}_W \mathbf{v}$ and find the distance from $\mathbf{v}$ to $W$.



______ / 15

**6.** (9 Points) Label the following statements as true or false. Justify your answers. (If true, cite appropriate facts or theorems. If false, explain why the opposite must be true or give a counterexample that shows why the statement is not necessarily true).

______ (i) If $A$ is a matrix whose columns are orthonormal, the linear mapping $\mathbf{x} \mapsto A\mathbf{x}$ preserves length.

______ (ii) If $\lambda + 5$ is a factor of the characteristic polynomial of $A$, then 5 is an eigenvalue of $A$.

______ (iii) If $A$ is a square matrix with $\det A = 0$, then one column of $A$ is a multiple of another column of $A$.



______ / 9

**7.** (16 Points) Let $A$ be an $n \times n$ matrix which is diagonalizable and whose eigenvalues are all nonnegative (i.e., $\geq 0$.) Show there exists an $n \times n$ matrix $B$ such that $B^2 = A$.
*Reminder:* In these proof-type questions the question is not answered by picking a particular matrix that satisfies the hypotheses. You need to make an argument which "works" for *any* matrix satisfying the hypotheses.



______ / 16

(This page intentionally left blank.)