

$$12. f(x,y) = x^2 + y^2 + \frac{1}{x^2 y^2}$$

$$f_x(x,y) = 2x - 2x^{-3}y^{-2}$$

$$f_y(x,y) = 2y - 2y^{-3}x^{-2}$$

Critical points at  $(1,1)$ ,  $(1,-1)$ ,  $(-1,1)$ , and  $(-1,-1)$

	evaluated at	$(1,1)$	$(1,-1)$	$(-1,1)$	$(-1,-1)$
$f_{xx} = 2 + 6x^{-4}y^{-2}$		8	8	8	8
$f_{yy} = 2 + 6x^2y^{-4}$		8	8	8	8
$f_{xy} = f_{yx} = 4x^{-3}y^{-3}$		4	-4	-4	4

$$D = \begin{array}{cccc} \begin{vmatrix} 8 & 4 \\ 4 & 8 \end{vmatrix} & \begin{vmatrix} 8 & -4 \\ -4 & 8 \end{vmatrix} & \begin{vmatrix} 8 & -4 \\ -4 & 8 \end{vmatrix} & \begin{vmatrix} 8 & 4 \\ 4 & 8 \end{vmatrix} \\ \parallel & \parallel & \parallel & \parallel \\ 48 & 48 & 48 & 48 \end{array}$$

$(1,1)$ ,  $(1,-1)$ ,  $(-1,1)$  and  $(-1,-1)$  are all local mins because  $f_{xx} > 0$ .

$$13. f(x,y) = x \sin y$$

$$f_x = \sin y$$

$$f_y = x \cos y$$

Critical points are at  $(0, k\pi)$  where  $k$  is an integer

evaluated at  $(0, k\pi)$

$$f_{xx} = 0$$

$$f_{yy} = -x \sin y$$

$$f_{xy} = f_{yx} = \cos y \leftarrow \begin{array}{l} 1 \text{ if } k \text{ is even} \\ -1 \text{ if } k \text{ is odd} \end{array}$$

So

$$D = \begin{vmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{vmatrix} = 0 - (\pm 1)^2 = -1 \quad \text{so these points are saddle points}$$