31. Shortest distance from $(2,1,-1)$ to plane $x+y-z=1$

$d=\sqrt{(x-2)^2+(y-1)^2+(z+1)^2}$ and we know that $z=x+y-1$

So we can minimize d by finding the absolute min of

$f(x,y)=(x-2)^2+(y-1)^2+(x+y)^2=x^2-4x+4+y^2-2y+1+x^2+2xy+y^2$

$f(x,y)=2x^2+2y^2-4x-2y+2xy+5$

$f_x=4x-4+2y$ find critical points: $4x-4+2y=0 \Rightarrow 2x-2+y=0$

$f_y=4y-2+2x$

$-(2x-2+4y=0)$

$-3y=0$

$y=0,\ x=1$

1.5 pt

The only critical point is $(1,0)$

$f_{xx}=4$

$f_{yy}=4$

$f_{xy}=f_{yx}=2$

$D=\begin{vmatrix}4 & 2\\ 2 & 4\end{vmatrix}=16-4=12$ $f_{xx}>0$, so local min.

So smallest $d=\sqrt{(1-2)^2+(0-1)^2+(1+0)^2}=\sqrt{(-1)^2+(-1)^2+(1)^2}=\sqrt{3}$

34. find points on $x^2y^2z=1$ closest to origin

$d=\sqrt{(x-0)^2+(y-0)^2+(z-0)^2} \Rightarrow d=\sqrt{x^2+y^2+z^2}$ and we know $z=\frac{1}{x^2y^2}$

1.5 p

minimize $f=\cancel{2}x^2+y^2+\frac{1}{x^4y^4}$

$f_x=2x-4x^{-5}y^{-4}$

$f_y=2y-4x^{-4}y^{-5}$

critical points $(2^{1/10},2^{1/10}),(-2^{1/10},2^{1/10}),(2^{1/10},-2^{1/10}),(-2^{1/10},-2^{1/10})$

evaluated at:

$f_{xx}=2+\frac{20}{x^6y^4}$	12	12	12	12
$f_{yy}=2+\frac{20}{x^6y^4}$	12	12	12	12
$f_{xy}=\frac{16}{x^5y^5}$	8	-8	-8	8

$D=\begin{vmatrix}12 & \pm 8\\ \pm 8 & 12\end{vmatrix}=144-64=80$, $f_{xx}>0$, so all are local mins.

So the points closest to origin are $(\pm 2^{1/10},\pm 2^{1/10},2^{-2/5})$