Math 20 - Spring 2002

Homework # 26 Solutions

Handout Section 3 / # 14, 17, 27, 35, 37, 49, 52, 54

14. $f(x,y) = x^5 + 3x^3y^2 + 3xy^4$

$$\frac{\partial f}{\partial x} = 5x^4 + 9x^2y^2 + 3y^4$$

$$\frac{\partial f}{\partial y} = 6x^3y + 12xy^3$$

17. $f(x,y) = \frac{x-y}{x+y}$

$$\frac{\partial f}{\partial x} = \frac{(x+y)-(x-y)}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{-(x+y)-(x-y)}{(x+y)^2} = \frac{-2x}{(x+y)^2}$$

27. $w = \ln(x+2y+3z)$

$$\frac{\partial w}{\partial x} = \frac{1}{x+2y+3z}$$

$$\frac{\partial w}{\partial y} = \frac{2}{x+2y+3z}$$

$$\frac{\partial w}{\partial z} = \frac{3}{x+2y+3z}$$

35. $f(x,y) = \sqrt{x^2+y^2}$

$$f_x(x,y) = \tfrac{1}{2}(x^2+y^2)^{-\frac{1}{2}}(2x)$$

$$= x(x^2+y^2)^{-1/2}$$

$$f_x(3,4) = 3\cdot(25)^{-1/2} = \frac{3}{5}$$

37. $f(x,y,z) = \frac{x}{y+z} = x(y+z)^{-1}$

$$f_z(x,y,z) = -x(y+z)^{-2}$$

$$f_z(3,2,1) = -3(2+1)^{-2} = -\frac{1}{3}$$

49. $u = e^{-s}\sin t$

$$\frac{\partial u}{\partial s} = -e^{-s}\sin t \qquad \frac{\partial u}{\partial t} = e^{-s}\cos t$$

$$\Rightarrow \frac{\partial^2 u}{\partial s^2} = e^{-s}\sin t \qquad \frac{\partial^2 u}{\partial t^2} = -e^{-s}\sin t$$

$$\frac{\partial^2 u}{\partial s\,\partial t} = -e^{-s}\cos t = \frac{\partial^2 u}{\partial t\,\partial s}.$$

52. $u = xye^y$

$$u_x = ye^y \qquad u_y = x[ye^y + e^y]$$

$$u_{xy} = ye^y + e^y \qquad u_{yx} = ye^y + e^y$$

So $u_{xy} = u_{yx}$.

54. $f(x,y) = e^{xy^2}$

$$f_x = y^2e^{xy^2}$$

$$f_{xx} = y^4e^{xy^2}$$

$$f_{xxy} = y^4(2xye^{xy^2}) + (e^{xy^2})(4y^3)$$

$$= 2xy^5e^{xy^2} + 4y^3e^{xy^2}$$