28. $T(x,y,z) = 200e^{-x^2-3y^2-9z^2}$.

(a) $\nabla T(x,y,z) = \left(-400xe^{-x^2-3y^2-9z^2},\ -1200ye^{-x^2-3y^2-9z^2},\ -3600ze^{-x^2-3y^2-9z^2}\right)$.

$\nabla T(2,-1,2) = \left(-800e^{-43},\ +1200e^{-43},\ -7200e^{-43}\right)$

The direction is $\vec{v} = (3,-3,3) - (2,-1,2) = (1,-2,1)$.

$\Rightarrow \vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{6}}(1,-2,1)$.

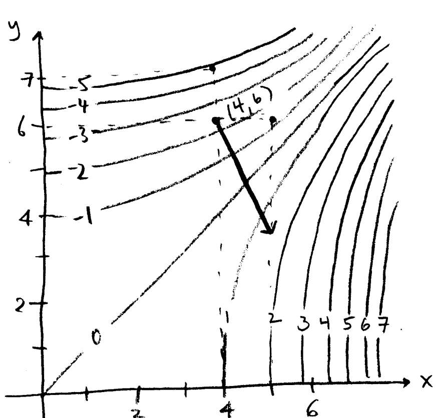
$$\Rightarrow D_{\vec{u}}T(2,-1,2) = \nabla T(2,1,2)\cdot\vec{u}$$
$$= \frac{1}{\sqrt{6}}\left[-800e^{-43} - 2400e^{-43} - 7200e^{-43}\right].$$
$$= -\frac{1}{\sqrt{6}}\,10400e^{-43}$$

(b) The temperature increases the fastest in the direction of the gradient at P (above).

(c) The maximal rate of increase of T is the magnitude of the gradient at P:

$$\|\nabla T(2,-1,2)\| = \sqrt{800^2 + 1200^2 + 7200^2}\cdot e^{-43}$$
$$\approx 7343e^{-43}.$$

34.



Approximate the gradient by approximating the partial derivatives.

$$\left.\frac{\partial f}{\partial x}\right|_{(4,6)} \approx 1$$

$$\left.\frac{\partial f}{\partial y}\right|_{(4,6)} \approx -2.5$$

This looks approximately correct since we know that the gradient should be $\perp$ to the level curves.