(18) WE NEED $\begin{bmatrix} A & 0 \\ B & A \end{bmatrix}\begin{bmatrix} A^{-1} & 0 \\ C & A^{-1} \end{bmatrix} = \begin{bmatrix} I_2 & 0 \\ 0 & I_2 \end{bmatrix}.$ CALCULATION

OF THE LEFT HAND SIDE YIELDS $\begin{bmatrix} I_2 & 0 \\ BA^{-1}+AC & I_2 \end{bmatrix}.$ SO WE MUST

HAVE $BA^{-1}+AC=0$, IE. $AC=-BA^{-1}$, IE $C=-A^{-1}BA^{-1}$

(19) A) HERE $A=\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ AND $0=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ SO $A^{-1}=\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

AND $C=-A^{-1}BA^{-1}=A^{-1}\left(\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\right)=\begin{bmatrix}0 & 0 \\ -1 & 0\end{bmatrix}.$ THUS, THE

INVERSE IS $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ -1 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

(23) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ IF AND ONLY IF

$a+g=1, \; b+h=0, \; c+i=0, \; a+d=0, \; b+e=1, \; c+f=0, \; d+g=0,$
$e+h=0, \; f+i=1.$ — USING THE FIRST, FOURTH AND SEVENTH

YIELDS $2a+\underbrace{d+g}_{0}=1 \Rightarrow a=\frac{1}{2}.$ So $d=-\frac{1}{2}$ AND $g=\frac{1}{2}.$

NOW USE THE 2ND, 5TH & 8TH EQNS TO GET $b=\frac{1}{2}, \; e=\frac{1}{2}, \; h=-\frac{1}{2},$

THEN USE THE 3RD, 6TH & 9TH EQNS TO GET $c=-\frac{1}{2}, \; f=\frac{1}{2}, \; i=\frac{1}{2}$

SO $A^{-1}=\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$