Solution Set 3.3

1.a. $u \cdot v = (2)(5) + (3)(-7) = -11$

d. $u \cdot v = (-2)(1) + (2)(7) + (3)(-4) = 0$

2a. $\cos\theta = \dfrac{u \cdot v}{\|u\|\|v\|} = \dfrac{-11}{\sqrt{13}\sqrt{74}}$

d. $\cos\theta = \dfrac{0}{\sqrt{17}\sqrt{66}} = 0$

3a. $u \cdot v = (6)(2) + (1)(0) + (4)(-3) = 0$, so they are orthogonal

d. $u \cdot v = (2)(5) + (4)(3) + (-8)(7) = -34$, so the angle is obtuse

} by thm 3.3.1

4a. $u \cdot a = 18 - 18 = 0$, so the orthogonal projection is $(0,0)$

d. $\text{Proj}_a u = \left(\dfrac{u \cdot a}{\|a\|^2}\right) a = \dfrac{4}{89}(4,3,8) = \left(\dfrac{16}{89}, \dfrac{12}{89}, \dfrac{32}{89}\right)$

5a. $w_2 = u - w_1 = (6,2) - (0,0) = (6,2)$

d. $w_2 = u - w_1 = (1,0,0) - \left(\frac{16}{89}, \frac{12}{89}, \frac{32}{89}\right) = \left(\dfrac{73}{89}, \dfrac{-12}{89}, \dfrac{-32}{89}\right)$

6a. Since $|u \cdot a| = |-4+6| = |2| = 2$, and $\|a\| = 5$, it follows from formula 10 that $\|\text{Proj}_a u\| = 2/5$.

d. Since $|u \cdot a| = |3 - 4 - 42| = |-43| = 43$, and $\|a\| = \sqrt{54}$, it follows from formula 10 that $\|\text{Proj}_a u\| = \dfrac{43}{\sqrt{54}}$

8.a. $(a,b) \cdot (-b,a) = -ab + ab = 0$ ✓

b. $(3,2)$ and $(-3,-2)$ are orthoganal to $(2,-3)$.

$(3)(2) + (2)(-3) = 0$

$(-3)(2) + (-2)(-3) = 0$ ✓

c. $(-4,-3)$ and $(4,3)$ are orthoganal, and $\|(4,3)\| = 5$.

So, orthogonal unit vectors would be $\left(\frac{-4}{5}, \frac{-3}{5}\right)$ and $\left(\frac{4}{5}, \frac{3}{5}\right)$.