

Math 20 - Spring 2002

Homework #18 Solutions

Total: 9 pts

7.1/1abc, 2abc, 4ab, 5ab, 10, 11, 14, 16, 20.

(1pt) ① (a)  $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} \Rightarrow \det(\lambda I - A) = 0 \Rightarrow \det \begin{bmatrix} \lambda - 3 & 0 \\ -8 & \lambda + 1 \end{bmatrix} = 0 \Rightarrow \boxed{(\lambda - 3)(\lambda + 1) = 0}$

(b)  $A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} \Rightarrow \det(\lambda I - A) = 0 \Rightarrow \det \begin{bmatrix} \lambda - 10 & 9 \\ -4 & \lambda + 2 \end{bmatrix} \Rightarrow (\lambda - 10)(\lambda + 2) + 36 = 0$   
 $\Rightarrow \boxed{\lambda^2 - 8\lambda + 16 = 0}$

(c)  $A = \begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix} \Rightarrow \det(\lambda I - A) = 0 \Rightarrow \det \begin{bmatrix} \lambda & -3 \\ -4 & \lambda \end{bmatrix} \Rightarrow \boxed{\lambda^2 - 12 = 0}$

② The eigenvalues are simply the roots of the characteristic equations found in ①

(a)  $\lambda = 3, -1$       (b)  $\lambda = 4$       (c)  $\lambda = \pm 2\sqrt{3}$

(1pt) ④ (a)  $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \Rightarrow \det(\lambda I - A) = 0 \Rightarrow \det \begin{bmatrix} \lambda - 4 & 0 & -1 \\ 2 & \lambda - 1 & 0 \\ 2 & 0 & \lambda - 1 \end{bmatrix} = 0$

$\Rightarrow (\lambda - 1)[(\lambda - 4)(\lambda - 1) + 2] = 0 \Rightarrow \boxed{(\lambda - 1)(\lambda - 3)(\lambda - 2) = 0}$

(b)  $A = \begin{bmatrix} 3 & 0 & -5 \\ 1/5 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix} \Rightarrow \det(\lambda I - A) = 0 \Rightarrow \det \begin{bmatrix} \lambda - 3 & 0 & 5 \\ -1/5 & \lambda + 1 & 0 \\ -1 & -1 & \lambda + 2 \end{bmatrix} = 0$

$\Rightarrow (\lambda - 3) \det \begin{bmatrix} \lambda + 1 & 0 \\ -1 & \lambda + 2 \end{bmatrix} + 5 \det \begin{bmatrix} -1/5 & \lambda + 1 \\ -1 & -1 \end{bmatrix} = 0$

$\Rightarrow (\lambda - 3)(\lambda + 1)(\lambda + 2) + 5(1/5 + (\lambda + 1)) = 0$

$\Rightarrow \boxed{\lambda^3 - 2\lambda = 0}$

(0.5pts) ⑤ (a)  $\lambda = 1, 3, 2$       (b)  $\lambda = 0, \pm\sqrt{2}$