SUMMARY OF SECTION 3.4

LET $\vec{U} = (U_1, U_2, U_3)$ AND $\vec{V} = (V_1, V_2, V_3)$. THEN

$$\vec{U} \times \vec{V} = \left(\begin{vmatrix} U_2 & U_3 \\ V_2 & V_3 \end{vmatrix}, -\begin{vmatrix} U_1 & U_3 \\ V_1 & V_3 \end{vmatrix}, \begin{vmatrix} U_1 & U_2 \\ V_1 & V_2 \end{vmatrix} \right)$$

NOTE: $\vec{U} \times \vec{V}$ IS A VECTOR PERPENDICULAR TO BOTH $\vec{U}$ AND $\vec{V}$.

THEOREM 1 – a) $\vec{U} \cdot (\vec{U} \times \vec{V}) = 0$ b) $\vec{V} \cdot (\vec{U} \times \vec{V}) = 0$

c) $\|\vec{U} \times \vec{V}\|^2 = \|\vec{U}\|^2 \|\vec{V}\|^2 - (\vec{U} \cdot \vec{V})^2$ d) $\vec{U} \times (\vec{V} \times \vec{W}) = (\vec{U} \cdot \vec{W})\vec{V} - (\vec{U} \cdot \vec{V})\vec{W}$

e) $(\vec{U} \times \vec{V}) \times \vec{W} = (\vec{U} \cdot \vec{W})\vec{V} - (\vec{V} \cdot \vec{W})\vec{U}$

THEOREM 2 – a) $\vec{U} \times \vec{V} = -\vec{V} \times \vec{U}$ b) $\vec{U} \times (\vec{V} + \vec{W}) = \vec{U} \times \vec{V} + \vec{U} \times \vec{W}$

c) $(\vec{U} + \vec{V}) \times \vec{W} = \vec{U} \times \vec{W} + \vec{V} \times \vec{W}$ d) $k(\vec{U} \times \vec{V}) = (k\vec{U}) \times \vec{V} = \vec{U} \times (k\vec{V})$

e) $\vec{U} \times \vec{0} = \vec{0} \times \vec{U} = \vec{0}$ f) $\vec{U} \times \vec{U} = \vec{0}$

WE LET $i = (1,0,0)$, $j = (0,1,0)$ AND $k = (0,0,1)$ AND NOTE
$i \times i = j \times j = k \times k = \vec{0}$, $i \times j = k$, $j \times k = i$, $k \times i = j$, $j \times i = -k$,
$k \times j = -i$ AND $i \times k = -j$.

NOTE THAT $\vec{U} \times \vec{V} = \begin{vmatrix} U_2 & U_3 \\ V_2 & V_3 \end{vmatrix} i - \begin{vmatrix} U_1 & U_3 \\ V_1 & V_3 \end{vmatrix} j + \begin{vmatrix} U_1 & U_2 \\ V_1 & V_2 \end{vmatrix} k$

$$= \begin{vmatrix} i & j & k \\ U_1 & U_2 & U_3 \\ V_1 & V_2 & V_3 \end{vmatrix}$$