

# SUMMARY OF SECTION 4.1

AN ORDERED N-TUPLE IS A SEQUENCE OF  $N$  REAL NUMBERS  $(A_1, A_2, \dots, A_N)$ . THE SET OF ALL ORDERED  $N$ -TUPLES IS CALLED  $N$ -SPACE AND IS DENOTED BY  $\mathbb{R}^N$ .

LET  $\vec{U} = (U_1, U_2, \dots, U_N)$  AND  $\vec{V} = (V_1, V_2, \dots, V_N)$ . THEN

$$\vec{U} = \vec{V} \iff U_1 = V_1, U_2 = V_2, \dots, U_N = V_N,$$

$$\vec{U} + \vec{V} = (U_1 + V_1, U_2 + V_2, \dots, U_N + V_N) \quad \& \quad k\vec{U} = (kU_1, kU_2, \dots, kU_N).$$

THE NEGATIVE OF  $\vec{U}$  IS DENOTED  $-\vec{U}$  AND IS DEFINED BY

$$-\vec{U} = (-U_1, -U_2, \dots, -U_N) \text{ SO THAT}$$

$$\vec{V} - \vec{U} = \vec{V} + (-\vec{U}) = (V_1 - U_1, V_2 - U_2, \dots, V_N - U_N).$$

THE ZERO VECTOR IS  $\vec{0} = (0, 0, \dots, 0)$ .

THM 1 - IF  $\vec{U}, \vec{V}$  AND  $\vec{W}$  ARE IN  $\mathbb{R}^N$  AND  $k \& L$  ARE SCALARS, THEN

$$\begin{aligned} \vec{U} + \vec{V} &= \vec{V} + \vec{U}, \quad \vec{U} + (\vec{V} + \vec{W}) = (\vec{U} + \vec{V}) + \vec{W}, \quad \vec{U} + \vec{0} = \vec{0} + \vec{U} = \vec{U}, \\ \vec{U} + (-\vec{U}) &= \vec{U} - \vec{U} = \vec{0}, \quad k(L\vec{U}) = (kL)\vec{U}, \quad k(\vec{U} + \vec{V}) = k\vec{U} + k\vec{V}, \\ (k+L)\vec{U} &= k\vec{U} + L\vec{U}, \quad 1 \cdot \vec{U} = \vec{U}. \end{aligned}$$

THE DOT PRODUCT OF  $\vec{U}$  AND  $\vec{V}$  IS  $\vec{U} \cdot \vec{V} = U_1V_1 + U_2V_2 + \dots + U_NV_N$ .

THM 2 -  $\vec{U} \cdot \vec{V} = \vec{V} \cdot \vec{U}$ ,  $(\vec{U} + \vec{V}) \cdot \vec{W} = \vec{U} \cdot \vec{W} + \vec{V} \cdot \vec{W}$ ,

$$(k\vec{U}) \cdot \vec{V} = k(\vec{U} \cdot \vec{V}), \quad \vec{V} \cdot \vec{V} \geq 0 \text{ AND } \vec{V} \cdot \vec{V} = 0 \iff \vec{V} = \vec{0}$$