SUMMARY OF SECTION 4.3

A LINEAR TRANSFORMATION IS SAID TO BE <u>ONE-TO-ONE</u> IF IT MAPS DISTINCT VECTORS INTO DISTINCT VECTORS.

<u>THEOREM 1</u> – IF A IS $N \times N$ AND $T_A : \mathbb{R}^N \to \mathbb{R}^N$ IS MULTIPLICATION BY A, THEN THE FOLLOWING ARE EQUIVALENT:

a) A IS INVERTIBLE b) THE RANGE OF T_A IS $\mathbb{R}^N$

c) T_A IS ONE TO ONE

IF A IS AN INVERTIBLE MATRIX, THEN THE INVERSE OF T_A IS $T_{A^{-1}}$. THAT IS TO SAY $T_A \circ T_{A^{-1}}(\vec{x}) = T_{A^{-1}} \circ T_A(\vec{x}) = \vec{x}$ FOR EACH $\vec{x}$ IN $\mathbb{R}^N$.

<u>THEOREM 2</u> – A FUNCTION $T : \mathbb{R}^N \to \mathbb{R}^m$ IS A LINEAR TRANSFORMATION $\iff$ FOR ALL $\vec{u}, \vec{v}$ IN $\mathbb{R}^N$ AND EACH SCALAR c

a) $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$ b) $T(c\vec{u}) = cT(\vec{u})$

LET $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ $\cdots$ $\vec{e}_N = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$

<u>THEOREM 3</u> – IF $T : \mathbb{R}^N \to \mathbb{R}^m$ IS A LINEAR TRANSFORMATION, THEN THE STANDARD MATRIX FOR T IS $\left[T(\vec{e}_1) \mid T(\vec{e}_2) \mid \cdots \mid T(\vec{e}_N) \right]$