SUMMARY OF SECTION 7.1

IF $A\vec{x} = \lambda\vec{x}$ FOR A NONZERO VECTOR $\vec{x}$, THEN λ IS CALLED AN <u>EIGENVALUE</u> OF A CORRESPONDING TO THE <u>EIGEN VECTOR</u> $\vec{x}$.

λ IS AN EIGENVALUE OF $A \iff \underbrace{\det(\lambda I - A) = 0}_{\text{CHARACTERISTIC EQUATION}}$.

IF A IS $N \times N$, $\det(\lambda I - A)$ IS AN N^{TH} DEGREE POLYNOMIAL IN λ CALLED THE <u>CHARACTERISTIC POLYNOMIAL</u>. SINCE SUCH A POLYNOMIAL HAS AT MOST N DISTINCT ROOTS, A CAN HAVE AT MOST N DISTINCT EIGENVALUES.

<u>THEOREM 1</u> – THE EIGENVALUES OF AN UPPER TRIANGULAR MATRIX ARE THE ENTRIES ON ITS MAIN DIAGONAL.

<u>THEOREM 2</u> – THE FOLLOWING STATEMENTS ARE EQUIVALENT:
λ IS AN EIGENVALUE OF A, $(\lambda I - A)\vec{x}$ HAS NONTRIVIAL SOLUTIONS, $A\vec{x} = \lambda\vec{x}$ FOR SOME NONZERO $\vec{x}$, $\det(\lambda I - A) = 0$.

<u>THEOREM 3</u> – IF λ IS AN EIGENVALUE OF A WITH EIGENVECTOR $\vec{x}$, THEN λ^k IS AN EIGENVECTOR OF A^k WITH EIGENVECTOR $\vec{x}$.
(k IS A POSITIVE INTEGER)

<u>THEOREM 4</u> – A IS INVERTIBLE $\iff$ 0 IS <u>NOT</u> AN EIGENVALUE OF A.