

SUMMARY OF SECTION 7.2

A SQUARE MATRIX A IS SAID TO BE DIAGONALIZABLE IF THERE IS AN INVERTIBLE MATRIX P SUCH THAT $P^{-1}AP$ IS DIAGONAL; THE MATRIX P IS SAID TO DIAGONALIZE A .

THM 1 - AN $N \times N$ MATRIX A IS DIAGONALIZABLE IF AND ONLY IF A HAS N EIGENVECTORS $\vec{p}_1, \vec{p}_2, \dots, \vec{p}_N$ SO THAT $[\vec{p}_1 | \vec{p}_2 | \dots | \vec{p}_N]$ IS INVERTIBLE.

NOTE: THE MATRIX $[\vec{p}_1 | \vec{p}_2 | \dots | \vec{p}_N]$ WILL DIAGONALIZE A .

THM 3 - IF AN $N \times N$ MATRIX A HAS N DISTINCT EIGENVALUES $\lambda_1, \lambda_2, \dots, \lambda_N$, THEN THE MATRIX OF CORRESPONDING EIGENVECTORS $[\vec{p}_1 | \vec{p}_2 | \dots | \vec{p}_N]$ IS INVERTIBLE.

NOTE: THM 2 & THM 4 DEAL WITH NOTATION FROM CH 5 & CH 6 AND ARE NOT NECESSARY FOR OUR PURPOSES.

COMPUTATIONAL TOOL \rightarrow IF $P^{-1}AP = D$ IS DIAGONAL, THEN $A^k = PD^kP^{-1}$ FOR ALL POSITIVE INTEGERS k .

FACT: AN $N \times N$ MATRIX A IS NOT DIAGONALIZABLE \Leftrightarrow THE TOTAL NUMBER OF PARAMETERS NEEDED TO DESCRIBE ITS EIGENVECTORS IS LESS THAN N .