SUMMARY OF SECTION 11.9

A SQUARE MATRIX E WITH NON-NEGATIVE ENTRIES IN WHICH THE SUM OF THE ENTRIES IN ANY COLUMN IS 1 IS CALLED AN EXCHANGE MATRIX. WE REGARD e_{ij} AS THE FRACTION OF THE TOTAL OUTPUT OF THE j^{TH} INDUSTRY PURCHASED BY THE i^{TH} INDUSTRY.

FACT - FOR ANY NxN EXCHANGE MATRIX E THERE IS A VECTOR

$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{pmatrix} = \vec{p}$$

WITH NON-NEGATIVE ENTRIES SUCH THAT $E\vec{p} = \vec{p}$.

WE REGARD p_i AS THE PRICE CHARGED BY INDUSTRY i FOR ITS TOTAL OUTPUT.

THM - IF E IS AN EXCHANGE MATRIX AND E^m HAS ALL POSITIVE ENTRIES FOR SOME POSITIVE INTEGER m, THEN THE SET OF ALL $\vec{p}$ SATISFYING $E\vec{p} = \vec{p}$ IS DESCRIBABLE WITH ONE PARAMETER. MOREOVER, THERE IS A SOLUTION OF $E\vec{p} = \vec{p}$ WITH ALL POSITIVE ENTRIES.

NOW SUPPOSE WE HAVE N INDUSTRIES WHICH HAVE TO SATISFY AN OUTSIDE DEMAND. LET d_i BE THE NUMBER OF DOLLARS OF OUTPUT DEMANDED OF THE i^{TH} INDUSTRY BY OUTSIDE INTERESTS. LET x_i BE THE TOTAL DOLLAR OUT PUT OF THE i^{TH} INDUSTRY.