THEOREM – THE REDUCED ROW-ECHELON FORM OF AN $N \times N$ MATRIX HAS A ROW OF ALL ZEROS, OR IS I_N.

IF A IS SQUARE, AND THERE IS A MATRIX B SO THAT $AB = BA = I$, THE A IS SAID TO BE <u>INVERTIBLE</u> AND B IS CALLED THE* INVERSE OF A. IF THERE IS NO SUCH B, A IS SAID TO BE <u>SINGULAR</u> *INVERSES ARE UNIQUE!

WE DENOTE THE INVERSE OF A BY A^{-1}.

THEOREM – IF $ad-bc \neq 0$, THEN THE INVERSE OF A IS

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad \underbrace{A}_{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}$$

THEOREM – A, B BOTH INVERTIBLE AND OF THE SAME SIZE $\Rightarrow (AB)^{-1} = B^{-1}A^{-1}$ (THIS CAN BE EXTENDED TO ANY # OF MATRICES)

IF A IS SQUARE, $A^0 = I$, $A^N = \underbrace{A \cdot A \cdots A}_{N \text{ TIMES}}$, $\underbrace{A^{-N} = (A^{-1})^N}_{\text{IF } A \text{ IS INVERTIBLE!}}$

THEOREM – $A^R A^S = A^{R+S}$ & $(A^R)^S = A^{RS}$ FOR INTEGERS R & S.

THEOREM – A INVERTIBLE $\Rightarrow (A^{-1})^{-1} = A$, $(A^N)^{-1} = (A^{-1})^N$ $N = 0, 1, 2, \ldots$

IF $k \neq 0$, k A SCALAR $(kA)^{-1} = \frac{1}{k}A^{-1}$

THEOREM – $(A^T)^T = A$, $(A+B)^T = A^T + B^T$, $(A-B)^T = A^T - B^T$

$(kA)^T = kA^T$ (k A SCALAR), $(AB)^T = B^T A^T$.

THEOREM – A INVERTIBLE $\Rightarrow (A^T)^{-1} = (A^{-1})^T$.