

$$\|\vec{u}\| = \sqrt{\vec{u} \cdot \vec{u}} = \sqrt{u_1^2 + u_2^2 + \dots + u_n^2}$$

$$d(u, v) = \|\vec{u} - \vec{v}\| = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + \dots + (u_n - v_n)^2}$$

THM 3 - $|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \|\vec{v}\|$

THM 4 - $\|\vec{u}\| \geq 0$, $\|\vec{u}\| = 0 \Leftrightarrow \vec{u} = \vec{0}$, $\|k\vec{u}\| = |k| \|\vec{u}\|$,

$$\|\vec{u} + \vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\| \leftarrow \text{TRIANGLE INEQUALITY}$$

THM 5 - $d(u, v) \geq 0$, $d(u, v) = 0 \Leftrightarrow u = v$, $d(u, v) = d(v, u)$

$$d(u, v) \leq d(u, w) + d(w, v) \leftarrow \text{TRIANGLE INEQUALITY}$$

THM 6 - $\vec{u} \cdot \vec{v} = \frac{1}{4} (\|\vec{u} + \vec{v}\|)^2 - \frac{1}{4} (\|\vec{u} - \vec{v}\|)^2$

\vec{u} AND \vec{v} ARE SAID TO BE ORTHOGONAL (PERPENDICULAR)

$$\Leftrightarrow \vec{u} \cdot \vec{v} = 0.$$

THM 7 - IF $\vec{u} \cdot \vec{v} = 0$, THEN $\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2$

IF \vec{u} AND \vec{v} ARE WRITTEN AS COLUMN MATRICES, THEN

$$\vec{u} \cdot \vec{v} = \vec{v}^T \vec{u} = \vec{u}^T \vec{v}. \text{ IF } A \text{ IS AN } N \times N \text{ MATRIX, THEN}$$

$$(A\vec{u}) \cdot \vec{v} = \vec{u} \cdot (A^T \vec{v}) \text{ AND } \vec{u} \cdot (A\vec{v}) = (A^T \vec{u}) \cdot \vec{v}$$