Math 20 - Spring 2002

Homework #22 Solutions

$\boxed{\text{Total: 6 pts}}$

11.18 / #1, 2, 3a, 4

1. (1.5 pts) $L = \begin{bmatrix} 1 & 3/2 \\ 1/2 & 0 \end{bmatrix}$

(a) $\det(\lambda I - L) = 0$: $\det\begin{bmatrix} \lambda-1 & -3/2 \\ -1/2 & \lambda \end{bmatrix} = 0 \Rightarrow \lambda(\lambda-1) - \frac{3}{4} = 0$

$$\lambda^2 - \lambda - \frac{3}{4} = 0.$$

$$(\lambda - \tfrac{3}{2})(\lambda + \tfrac{1}{2}) = 0.$$

$$\Rightarrow \lambda = \tfrac{3}{2}, -\tfrac{1}{2}.$$

The positive eigenvalue is $\boxed{\lambda_1 = \frac{3}{2}}$

E_1: $(\frac{3}{2}I - L) = \begin{bmatrix} 1/2 & -3/2 \\ -1/2 & 3/2 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix}$. $\Rightarrow$ $y_1 = 3y_2 = 3s$, $y_2 = s$

The corresponding eigenvector is $\boxed{x_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}}$

(b) $x^{(0)} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$

$\Rightarrow x^{(1)} = Lx^{(0)} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$ $\qquad x^{(2)} = Lx^{(1)} = \begin{bmatrix} 175 \\ 50 \end{bmatrix}$

$x^{(3)} = Lx^{(2)} = \begin{bmatrix} 250 \\ 87.5 \end{bmatrix} \approx \begin{bmatrix} 250 \\ 88 \end{bmatrix}$ $\qquad x^{(4)} = Lx^{(3)} = \begin{bmatrix} 382 \\ 125 \end{bmatrix}$

$x^{(5)} = Lx^{(4)} = \begin{bmatrix} 569.5 \\ 191 \end{bmatrix} \approx \begin{bmatrix} 570 \\ 191 \end{bmatrix}$

(c) $x^{(6)} = Lx^{(5)} = \begin{bmatrix} 856.5 \\ 285 \end{bmatrix} \approx \begin{bmatrix} 857 \\ 285 \end{bmatrix}$

$x^{(6)} \approx \lambda_1 x^{(5)} = \begin{bmatrix} 855 \\ 286.5 \end{bmatrix} \approx \begin{bmatrix} 855 \\ 287 \end{bmatrix}$