

$$6. f(x,y) = x^3y + 12x^2 - 8y$$

$$f_x = 3x^2y + 24x = 3x(xy + 8)$$

$$f_y = x^3 - 8$$

1 pt Only critical point is at  $(2, -4)$

$$f_{xx}(2, -4) = 6xy + 24 \Big|_{(2, -4)} = -24$$

$$f_{yy}(2, -4) = 0$$

$$f_{xy} = f_{yx} = 3x^2 \Big|_{(2, -4)} = 12$$

$$D = \begin{vmatrix} -24 & 12 \\ 12 & 0 \end{vmatrix} = -144 \therefore \text{neither local max or min.}$$

$$7. f(x,y) = x^2 + y^2 + x^2y + 4$$

$$f_x(x,y) = 2x + 2xy = 2x(1+y)$$

$$f_y(x,y) = 2y + x^2$$

Critical points:  $(0,0)$  and  $(\sqrt{2}, -1)$  and  $(-\sqrt{2}, -1)$

evaluated at	$(0,0)$	$(\sqrt{2}, -1)$	$(-\sqrt{2}, -1)$
$f_{xx} = 2 + 2y$	2	0	0
$f_{yy} = 2$	2	2	2
$f_{xy} = f_{yx} = 2x$	0	$2\sqrt{2}$	$-2\sqrt{2}$

$$D \quad \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \quad \begin{vmatrix} 0 & 2\sqrt{2} \\ 2\sqrt{2} & 2 \end{vmatrix} \quad \begin{vmatrix} 0 & -2\sqrt{2} \\ -2\sqrt{2} & 2 \end{vmatrix}$$

$$= 4$$

$$= -8$$

$$= -8$$

So the only ~~critical~~ <sup>local max or min</sup> point is  $(0,0)$ ; it's a local min because  $f_{xx} > 0$ .