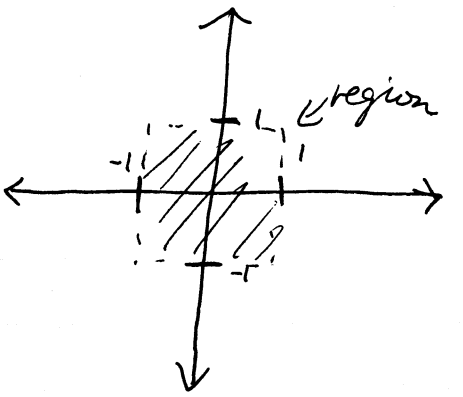
25)



absolute max and min of f on D

$D = \{(x,y) \mid |x| \le 1, |y| \le 1\}$

$f(x,y) = x^2 + y^2 + x^2y + 4$

$f_x = 2x + 2xy = 2x(1+y)$

$f_y = 2y + x^2$

Critical points in region are $(0,0)$ and ~~$(\sqrt{2},-1)$~~ ← out of range ~~$(-\sqrt{2},-1)$~~

$f_{xx}(0,0) = 2 + 2(0) = 2$

$f_{yy}(0,0) = 2$

$f_{xy} = f_{yx} = 2(0) = 0$

$D = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$

Since $f_{xx} > 0$, $(0,0)$ is a local min.

2 pt

$f(0,0) = 4$, but is this the absolute min?

Check the boundary (also, the max is clearly on the boundary).

		max	min
By:	$f(x,1) = x^2 + 1 + x^2(1) + 4 = 2x^2 + 5$	$x = 1, -1$; $f = 7$	$x = 0$; $f = 5$
	$f(x,-1) = x^2 + 1 - x^2 + 4 = 5$	$f = 5$	$f = 5$
	$f(1,y) = 1 + y^2 + (1)y + 4 = y^2 + y + 5$	$y = 1$, $f = 7$	$y = -\frac{1}{2}$, $f = 4.75$

$\frac{df}{dy} = 2y + 1$ $y = -\frac{1}{2}$ crit point

$\frac{d^2f}{dy^2} = 2$, so local min or at $y = 1, -1$

$f(-1,y) = 1 + y^2 + y + 4 = y^2 + y + 5$ $\quad y = 1$, $f = 7$ $\quad y = -\frac{1}{2}$, $f = 4.75$

So, the absolute min is at $(0,0)$, where $f = 4$

the absolute max is reached at two points: $(1,1)$ and $(-1,1)$ where $f = 7$