Math 20 - Spring 2002

Homework # 5 Solutions

1.6/ # 2, 3, 5, 8, 14, 17, 21, 22.

$\boxed{\text{Total: 10pts}}$

(2) (1pt)
$$\left.\begin{aligned} 4x_1 - 3x_2 &= -3 \\ 2x_1 - 5x_2 &= 9 \end{aligned}\right\} \Rightarrow \underbrace{\begin{bmatrix} 4 & -3 \\ 2 & -5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -3 \\ 9 \end{bmatrix}}_{b}$$

If A^{-1} exists $\Rightarrow$ $x = A^{-1}b$.

$$A^{-1} = \frac{1}{-20+6} \begin{bmatrix} -5 & 3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 5/14 & -3/14 \\ 2/14 & -4/14 \end{bmatrix} \Rightarrow$$

$$\boxed{x = A^{-1}b = \begin{bmatrix} 5/14 & -3/14 \\ 1/7 & -2/7 \end{bmatrix} \begin{bmatrix} -3 \\ 9 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}}$$

(3) (1pt)
$$\left.\begin{aligned} x_1 + 3x_2 + x_3 &= 4 \\ 2x_1 + 2x_2 + x_3 &= -1 \\ 2x_1 + 3x_2 + x_3 &= 3 \end{aligned}\right\} \Rightarrow \underbrace{\begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}}_{b}$$

Using the methods of Section 1.5, we obtain $A^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$

So
$$\boxed{x = A^{-1}b = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}}$$

(5) (1pt)
$$\left.\begin{aligned} x + y + z &= 5 \\ x + y - 4z &= 10 \\ -4x + y + z &= 0 \end{aligned}\right\} \Rightarrow \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -4 \\ -4 & 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}}_{b}$$

Using the methods of Section 1.5, we obtain $A^{-1} = \begin{bmatrix} 1/5 & 0 & -1/5 \\ 3/5 & 1/5 & 1/5 \\ 1/5 & -1/5 & 0 \end{bmatrix}$

So
$$\boxed{x = A^{-1}b = \begin{bmatrix} 1/5 & 0 & -1/5 \\ 3/5 & 1/5 & 1/5 \\ 1/5 & -1/5 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}}$$

(8) (1pt)
$$\left.\begin{aligned} x_1 + 2x_2 + 3x_3 &= b_1 \\ 2x_1 + 5x_2 + 5x_3 &= b_2 \\ 3x_1 + 5x_2 + 8x_3 &= b_3 \end{aligned}\right\} \Rightarrow \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 5 & 8 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{b}$$

$$A^{-1} = \begin{bmatrix} -15/2 & 1/2 & 5/2 \\ 1/2 & 1/2 & -1/2 \\ 5/2 & -1/2 & -1/2 \end{bmatrix}$$

$$\Rightarrow \boxed{x = A^{-1}b = \begin{bmatrix} -15/2 & 1/2 & 5/2 \\ 1/2 & 1/2 & -1/2 \\ 5/2 & -1/2 & -1/2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} -\frac{15}{2}b_1 + \frac{1}{2}b_2 + \frac{5}{2}b_3 \\ \frac{1}{2}b_1 + \frac{1}{2}b_2 - \frac{1}{2}b_3 \\ \frac{5}{2}b_1 - \frac{1}{2}b_2 - \frac{1}{2}b_3 \end{bmatrix}}$$