(14) (2 pts)

$$\left.\begin{aligned} x_1 + 3x_2 + 5x_3 &= b_1 \\ -x_1 - 2x_2 \quad &= b_2 \\ 2x_1 + 5x_2 + 4x_3 &= b_3 \end{aligned}\right\} \Rightarrow \underbrace{\begin{bmatrix} 1 & 3 & 5 \\ -1 & -2 & 0 \\ 2 & 5 & 4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{b} \qquad A^{-1} = \begin{bmatrix} 8 & -13 & -10 \\ -4 & 6 & 5 \\ 1 & -1 & -1 \end{bmatrix}$$

$\Rightarrow x = A^{-1}b$

(a) $b_1 = 1,\ b_2 = 0,\ b_3 = -1$ $\Rightarrow$ $\boxed{x = A^{-1}b = \begin{bmatrix} 8 & -13 & -10 \\ -4 & 6 & 5 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 18 \\ -9 \\ 2 \end{bmatrix}}$

(b) $b_1 = 0,\ b_2 = 1,\ b_3 = 1$ $\Rightarrow$ $\boxed{x = A^{-1}b = \begin{bmatrix} 8 & -13 & -10 \\ -4 & 6 & 5 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -23 \\ 11 \\ -2 \end{bmatrix}}$

(c) $b_1 = -1,\ b_2 = -1,\ b_3 = 0$ $\Rightarrow$ $\boxed{x = A^{-1}b = \begin{bmatrix} 8 & -13 & -10 \\ -4 & 6 & 5 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}}$

(17) (1 pt)

$$\begin{aligned} x_1 - 2x_2 + 5x_3 &= b_1 \\ 4x_1 - 5x_2 + 8x_3 &= b_2 \\ -3x_1 + 3x_2 - 3x_3 &= b_3 \end{aligned}$$

For what b_1, b_2, b_3 is this system consistent?

$\Downarrow$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 4 & -5 & 8 & b_2 \\ -3 & 3 & -3 & b_3 \end{array}\right] \xrightarrow[3R_1 + R_3 \to R_3']{-4R_1 + R_2 \to R_2'} \left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & -4b_1 + b_2 \\ 0 & -3 & 12 & 3b_1 + b_3 \end{array}\right] \xrightarrow[-\frac{1}{3}R_3 \to R_3']{\frac{1}{3}R_2 \to R_2'} \left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 1 & -4 & -\frac{4}{3}b_1 + \frac{1}{3}b_2 \\ 0 & 1 & -4 & -b_1 - \frac{1}{3}b_3 \end{array}\right]$$

$$\xrightarrow{-R_2 + R_3 \to R_3'} \left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 1 & -4 & -\frac{4}{3}b_1 + \frac{1}{3}b_2 \\ 0 & 0 & 0 & \frac{1}{3}b_1 - \frac{1}{3}b_2 - \frac{1}{3}b_3 \end{array}\right]$$

For the system to be consistent, $\frac{1}{3}(b_1 - b_2 - b_3) = 0 \Rightarrow b_1 - b_2 = b_3$.

So in order for this system to be consistent, we need b of the form $\boxed{b = \begin{bmatrix} b_1 \\ b_2 \\ b_1 - b_2 \end{bmatrix}}$

(21) (1 pt)

$$\underbrace{\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}}_{A} X = \underbrace{\begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}}_{B}$$

Since A is 3×3 and B is 3×5, X must be 3×5.

If A^{-1} exists, then $AX = B \Rightarrow A^{-1}AX = A^{-1}B \Rightarrow X = A^{-1}B$.

Using the methods from Section 1.5, we obtain $A^{-1} = \begin{bmatrix} 3 & -1 & 3 \\ -2 & 1 & -2 \\ -4 & 2 & -5 \end{bmatrix}$

So $\boxed{X = A^{-1}B = \begin{bmatrix} 3 & -1 & 3 \\ -2 & 1 & -2 \\ -4 & 2 & -5 \end{bmatrix}\begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 12 & -3 & 27 & 26 \\ -6 & -8 & 1 & -18 & -17 \\ -15 & -21 & 9 & -38 & -35 \end{bmatrix}}$