

11.4 1, 2, 9, 10, 13, 20, 22, 24, 30 1pt each, 9 pts total

1. $z = 4x^2 - y^2 + 2y$; $(-1, 2, 4)$

$$f_x = 8x \quad f_y = -2y + 2$$

$$f_x(-1, 2) = -8 \quad f_y(-1, 2) = -2$$

Tangent plane: $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

$$z - 4 = -8(x + 1) - 2(y - 2)$$

$$\boxed{z = -8x - 2y}$$

2. $e^{x^2 - y^2} = z$, $(1, -1, 1)$

$$f_x = 2xe^{x^2 - y^2} \quad f_y = -2ye^{x^2 - y^2}$$

$$f_x(1, -1) = 2$$

$$f_y(1, -1) = 2$$

Tangent plane: $z - 1 = 2(x - 1) + 2(y + 1)$

$$\boxed{z = 2x + 2y + 1}$$

9. $f(x, y) = x\sqrt{y}$, $(1, 4)$ $f(1, 4) = 2$

f is differentiable at $(1, 4)$ if f_x, f_y exist and are continuous at $(1, 4)$.

$$f_x = \sqrt{y} \quad f_y = \frac{x}{2\sqrt{y}}$$

$$f_x(1, 4) = 2$$

$$f_y(1, 4) = \frac{1}{4}$$

So both partials exist + are continuous.

$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$\boxed{L(x, y) = 2 + 2(x - 1) + \frac{1}{4}(y - 4) = 2x + \frac{1}{4}y - 1}$$

10. $f(x, y) = x/y$, $(6, 3)$ $f(6, 3) = 2$

Again, we need the partials to exist and be continuous at $(6, 3)$

$$f_x = \frac{1}{y} \quad f_y = -\frac{x}{y^2}$$

$$f_x(6, 3) = \frac{1}{3} \quad f_y(6, 3) = -\frac{6}{9} = -\frac{2}{3}$$

$$\boxed{L(x, y) = 2 + \frac{1}{3}(x - 6) - \frac{2}{3}(y - 3) = \frac{1}{3}x - \frac{2}{3}y + 2}$$