SOLUTIONS TO PROBLEMS FROM HOMEWORK 8

2.3 #2, 3, 5, 7, 9, 14, 15, 16

(2)

$$\det(A) = \begin{vmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 \\ 0 & \frac{5}{2} & 0 \\ 0 & 0 & 2 \end{vmatrix} = 2\left(\tfrac{5}{2}\right)(2) = 10$$

add $-\frac{3}{2}$ (1) to (2)

$$\det(B) = \begin{vmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 3 \\ 0 & 8 & -19 \\ 0 & 5 & -14 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 3 \\ 0 & 8 & -19 \\ 0 & 0 & -\frac{17}{8} \end{vmatrix} = (1)(8)\left(-\tfrac{17}{8}\right) = -17$$

add -7 (1) to (2) add $-\frac{5}{8}$ (2) to (3)

add -5 (1) to (3)

$$\det(AB) = \det\left(\begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}\right) = \begin{vmatrix} 9 & -1 & 8 \\ 31 & 1 & 17 \\ 10 & 0 & 2 \end{vmatrix} = \begin{array}{l} (9)(1)(2) - 9(17)(0) - (-1)(31)(2) \\ + (-1)(17)(10) + 8(31)(0) - 8(1)(10) \\ = -170 \end{array}$$

$\det(A)\det(B) = (10)(-17) = -170 = \det(AB)$ ✓✓

(3)

$$A = \begin{bmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{bmatrix}$$

Note that (1) + (2) is equal to (3). So we know that (3) can be written as a linear combination of (1) and (2), and if we row reduce we get a row of zeros. Thus, det(A) is equal to zero.

(5)

$$\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -7$$

a) $\det(3A) = 3^3 \det(A) = (27)(-7) = \boxed{-189}$

b) $\det(A^{-1}) = \dfrac{1}{\det(A)} = \boxed{\dfrac{1}{-7}}$

c) $\det(2A^{-1}) = 2^3 \det(A^{-1}) = \dfrac{8}{\det(A)} = \boxed{\dfrac{8}{-7}}$

d) $\det((2A)^{-1}) = \dfrac{1}{\det(2A)} = \dfrac{1}{2^3\det(A)} = \dfrac{1}{8(-7)} = \boxed{\dfrac{1}{-56}}$

e) $\det\begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix} = \det\begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix} = (-1)\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = (-1)(-7) = \boxed{7}$