Math 20 3.2 Solutions.

1. a. $\|v\| = \sqrt{4^2+(-3)^2} = 5$ b. $\|v\| = \sqrt{2^2+3^2} = \sqrt{13}$

d. $\|v\| = \sqrt{2^2+2^2+2^2} = 2\sqrt{3}$ e. $\|v\| = \sqrt{(-7)^2+2^2+(-1)^2} = \sqrt{54} = 3\sqrt{6}$

2. b. $d = \sqrt{(-4-6)^2+(-1+3)^2} = \sqrt{100+4} = 2\sqrt{26}$

c. $d = \sqrt{(-7-7)^2+(-2+5)^2+(-1-1)^2} = \sqrt{196+9+4} = \sqrt{209}$

3. a. $u+v = (3,-5,7)$ so, $\|u+v\| = \sqrt{3^2+(-5)^2+7^2} = \sqrt{83}$

b. $\|u\| + \|v\| = \sqrt{2^2+(-2)^2+3^2} + \sqrt{1^2+(-3)^2+4^2} = \sqrt{17} + \sqrt{26}$

c. $\|-2u\| + 2\|u\| = \sqrt{(-4)^2+4^2+(-6)^2} + 2\sqrt{2^2+(-2)^2+3^2} = 2\sqrt{17} + 2\sqrt{17} = 4\sqrt{17}$

d. $\|3u-5v+w\| = \|(4,15,-15)\| = \sqrt{4^2+15^2+(-15)^2} = \sqrt{16+450} = \sqrt{466}$

e. since $\|w\| = \sqrt{3^2+6^2+(-4)^2} = \sqrt{61}$, then $\frac{1}{\|w\|}\cdot w = \left(\frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}}, \frac{-4}{\sqrt{61}}\right)$.

f. $\left\|\frac{1}{\|w\|}\cdot w\right\| = \left\|\left(\frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}}, \frac{-4}{\sqrt{61}}\right)\right\| = \sqrt{\left(\frac{3}{\sqrt{61}}\right)^2+\left(\frac{6}{\sqrt{61}}\right)^2+\left(\frac{-4}{\sqrt{61}}\right)^2} = \sqrt{\frac{9}{61}+\frac{36}{61}+\frac{16}{61}} = 1$

4. since $kv = (-k, 2k, 5k)$ then $\|kv\| = \sqrt{k^2+4k^2+25k^2} = |k|\sqrt{30}$.

If $\|kv\| = 4$, then $k = \pm\frac{4}{\sqrt{30}}$

5. a. Show $(u+v)+w = u+(v+w)$

$$((7,-3,1)+(9,6,6)) + (2,1,-8) = (7,-3,1) + ((9,6,6)+(2,1,-8))$$
$$(16,3,7)+(2,1,-8) = (7,-3,1)+(11,7,-2)$$
$$(18,4,-1) = (18,4,-1) \checkmark$$

b. Show $k(\ell u) = (k\ell)u$

$$-2(5\cdot(7,-3,1)) = (-2\cdot 5)\cdot(7,-3,1)$$
$$-2(35,-15,5) = (-10)(7,-3,1)$$
$$(-70,30,-10) = (-70,30,-10) \checkmark$$

6. a. If w is a unit vector, then $\|w\| = 1$.

$v = (v_1, v_2, v_3)$ for nonzero v.

$\|v\| = \sqrt{v_1^2+v_2^2+v_3^2}$

$$\frac{v}{\|v\|} = \left(\frac{v_1}{\sqrt{v_1^2+v_2^2+v_3^2}}, \frac{v_2}{\sqrt{v_1^2+v_2^2+v_3^2}}, \frac{v_3}{\sqrt{v_1^2+v_2^2+v_3^2}}\right)$$

$$\left\|\frac{v}{\|v\|}\right\| = \sqrt{\frac{v_1^2}{v_1^2+v_2^2+v_3^2}+\frac{v_2^3}{v_1^2+v_2^2+v_3^2}+\frac{v_3^2}{v_1^2+v_2^2+v_3^2}} = \sqrt{\frac{v_1^2+v_2^2+v_3^2}{v_1^2+v_2^2+v_3^2}} = 1.$$