15. by Theorem 3.4.2

$$(\vec{u}+\vec{v}) \times (\vec{u}-\vec{v}) = \vec{u} \times (\vec{u}-\vec{v}) + \vec{v} \times (\vec{u}-\vec{v})$$

$$= (\vec{u} \times \vec{u}) + (\vec{u} \times (-\vec{v})) + (\vec{v} \times \vec{u}) + (\vec{v} \times (-\vec{v}))$$

$$= \vec{0} - (\vec{u} \times \vec{v}) - (\vec{u} \times \vec{v}) - \vec{0}$$

$$= -2(\vec{u} \times \vec{v})$$

20. $\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cos\theta$ and

$\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \cdot \|\vec{v}\| \sin\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\|\vec{u} \times \vec{v}\|}{\|\vec{u}\| \, \|\vec{v}\|} \Big/ \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|} = \frac{\|\vec{u} \times \vec{v}\|}{\vec{u} \cdot \vec{v}}$$

24. a) $(\vec{u} + k\vec{v}) \times \vec{v} = (\vec{u} \times \vec{v}) + (k\vec{v} \times \vec{v})$

$$= (\vec{u} \times \vec{v}) + k(\vec{v} \times \vec{v})$$

$$= (\vec{u} \times \vec{v}) + k\vec{0}$$

$$= \vec{u} \times \vec{v}$$

b) Let $\vec{u} = (u_1, u_2, u_3)$ $\vec{v} = (v_1, v_2, v_3)$

$\vec{z} = (z_1, z_2, z_3)$

$$\vec{u} \cdot (\vec{v} \times \vec{z}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$$

since $\vec{x} \cdot \vec{y} = \vec{y} \cdot \vec{x}$ for any vectors X and y

$(\vec{u} \times \vec{z}) \cdot \vec{v} = \vec{v} \cdot (\vec{u} \times \vec{z})$