Section 4.2 Solution Set   1, 3, 6d, 9, 11, 13, 17, 21

1. a. $(x_1, x_2, x_3) \rightarrow (w_1, w_2)$, so $\mathbb{R}^3 \rightarrow \mathbb{R}^2$ and linear
   b. $(x_1, x_2) \rightarrow (w_1, w_2, w_3)$, so $\mathbb{R}^2 \rightarrow \mathbb{R}^3$, not linear bc of $2x_1 x_2$ and $3x_1$ ter
   c. $(x_1, x_2, x_3) \rightarrow (w_1, w_2, w_3)$, so $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ and linear
   d. $(x_1, x_2, x_3, x_4) \rightarrow (w_1, w_2)$, so $\mathbb{R}^4 \rightarrow \mathbb{R}^2$, not linear bc of x_1^2 and x_3^2

3. The standard matrix is A, where

$$w = Ax = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

so that $T(-1,2,4) = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$

6d.
$$T(x) = \begin{bmatrix} -1 & 1 \\ 2 & 4 \\ 7 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 \\ 2x_1 + 4x_2 \\ 7x_1 + 8x_2 \end{bmatrix}$$

9. a.
$$T(2,-5,3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}$$

b.
$$T(2,-5,3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

c.
$$T(2,-5,3) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -5 \\ 3 \end{bmatrix}$$

11a.
$$T(-2,1,3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

b.
$$T(-2,1,3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$$

c.
$$T(-2,1,3) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$