4.3 Solutions

1, 4, 6ad, 8ad, 14, 16, 18ad, 20 (1 point each; 8 pts total)

1. a) projections are not one-to-one because two different vectors can have the same projection.

b) Since a reflection is its own inverse, it is a one-to-one mapping.

4. by thm 4.3.1 we know that a linear operator has range of $\mathbb{R}^n$ if and only if the matrix representation is invertible.

$T_A: \mathbb{R}^3 \to ?$

$w_1 = x_1 - 2x_2 + x_3$

$w_2 = 5x_1 - x_2 + 3x_3$

$w_3 = 4x_1 + x_2 + 2x_3$

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 5 & -1 & 3 \\ 4 & 1 & 2 \end{bmatrix}$$

$\det A = 1(-2-3) + 2(10-12) + 1(5+4) = 0$

so A is not invertible.

$\therefore$ the range is not $\mathbb{R}^3$.

6.a) $[T] = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is invertible, so T is one-to-one.

$$[T]^{-1} = \begin{bmatrix} 1 & -2 & 4 \\ -1 & 2 & -3 \\ -1 & 3 & -5 \end{bmatrix} \quad \text{so} \quad T^{-1}(w_1, w_2, w_3)$$

$$= \begin{bmatrix} 1 & -2 & 4 \\ -1 & 2 & -3 \\ -1 & 3 & -5 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = (w_1 - 2w_2 + 4w_3, -w_1 + 2w_2 - 3w_3, -w_1 + 3w_2 - 5w_3)$$

d) Not one-to-one b/c

$$\det \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 4 \\ 1 & 4 & -5 \end{bmatrix} = 1(-5-16) - 2(10-28) + 1(-8-7) = 0$$