

20. a) Yes. If T_1, T_2 are one-to-one each maps distinct vectors to distinct vectors. Therefore $T_1 \circ T_2$ is one-to-one.

b) If T_2 is one-to-one and T_1 is not, $T_2 \circ T_1$ cannot be. There exist vectors \vec{u}, \vec{v} such that $T_1(\vec{u}) = T_1(\vec{v})$. Thus $T_2 \circ T_1(\vec{u}) = T_2 \circ T_1(\vec{v})$ and $T_2 \circ T_1$ is not one-to-one.

However, $T_1 \circ T_2$ may be one-to-one.

example: $[T_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $[T_2] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$[T_1 \circ T_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which clearly is one-to-one.