(10) (a) $A = \begin{bmatrix} -1 & 6 \\ 0 & 5 \end{bmatrix}$ $\Rightarrow$ eigenvalues = diagonal entries $\Rightarrow$ $\boxed{\lambda = 1, -5}$

(1pt)

(b) $A = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}$ $\Rightarrow$ eigenvalues = diagonal entries $\Rightarrow$ $\boxed{\lambda = 3, 7, 1}$

(c) $A = \begin{bmatrix} -1/3 & 0 & 0 & 0 \\ 0 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$ $\Rightarrow$ $\boxed{\lambda = -\frac{1}{3}, -\frac{1}{3}, 1, \frac{1}{2}}$

(11) Find eigenvalues of A^9 where $A = \begin{bmatrix} 1 & 3 & 7 & 11 \\ 0 & \frac{1}{2} & 3 & 8 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

(1.5pts)

Note that the eigenvalues of A are simply the diagonal entries of A:

$\Rightarrow \lambda = 1, \frac{1}{2}, 0, 2$. We want to find λ' s.t. $A^9x = \lambda'x$.

We have that $A^9x = A^8Ax = A^8 \cdot \lambda x = \lambda A^8 x$. where λ is an eigenvalue of A. Proceeding in this fashion we obtain $A^9x = \lambda^9 x$.

So $\lambda' = \lambda^9$. $\Rightarrow$ $\boxed{\lambda' = 1, (\tfrac{1}{2})^9, 0, (2)^9}$

(14) (a) $p(\lambda) = \lambda^3 - 2\lambda^2 + \lambda + 5$.

(1pt)

We know $\det(\lambda I - A) = \lambda^3 - 2\lambda + \lambda + 5$.

Setting $\lambda = 0 \Rightarrow \det(0 \cdot I - A) = 5$ $\Rightarrow$ $\det(-A) = 5 \Rightarrow \boxed{\det(A) = (-1)^n 5}$

where A is $n \times n$.

(b) $p(\lambda) = \lambda^4 - \lambda^3 + 7$.

$\det(\lambda I - A) = \lambda^4 - \lambda^3 + 7$.

$\lambda = 0 \Rightarrow \det(-A) = 7$ $\Rightarrow$ $\boxed{\det(A) = (-1)^n 7}$