(1b) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. $\det(\lambda I - A) = 0$

(1pt)

$\Rightarrow \det \begin{bmatrix} \lambda - a & -b \\ -c & \lambda - d \end{bmatrix} = 0 \Rightarrow (\lambda - a)(\lambda - d) - bc = 0$

$\Rightarrow \lambda^2 - \underbrace{(a+d)}_{tr(A)} + \underbrace{(ad - bc)}_{det(A)} = 0$.

$\Rightarrow \lambda^2 - tr(A) + det(A) = 0$. $\blacksquare$

(2b) Show: λ eigenvalue of A, invertible $\vec{x}$ corresponding eigenvector

(1.5pts) $\Rightarrow \frac{1}{\lambda}$ eigenvalue of A^{-1}, $\vec{x}$ is corresponding eigenvector

Pf: If λ is eigenvalue of A w/ $\vec{x}$ corresponding eigenvector, by def'n

$$Ax = \lambda x.$$

A is invertible, so multiply through on the left by A^{-1}:

$$A^{-1}Ax = A^{-1}(\lambda x)$$
$$x = \lambda A^{-1}x.$$

Divide by λ on both sides:

$$\frac{1}{\lambda}x = A^{-1}x.$$

This is of the form $Bx = \mu x$. So $\frac{1}{\lambda}$ is an eigenvalue of A w/ $\vec{x}$ corresponding eigenvector $\blacksquare$