Solutions to HW 19 Tanner Fahl

Section 7.2 # 8, 10, 12, 13, 19, 21

$\boxed{\text{TOTAL 9 PTS.}}$

⑧

$$\det(\lambda I - A) = \begin{vmatrix} \lambda+14 & -12 \\ 20 & \lambda-17 \end{vmatrix} = (\lambda+14)(\lambda-17)+12(20) = \lambda^2-3\lambda-238+240 = \lambda^2-3\lambda+2$$

$\boxed{\text{1.5 pts}}$

$$\lambda^2-3\lambda+2 = (\lambda-2)(\lambda-1) = 0 \Rightarrow \lambda_1 = 1 \text{ and } \lambda_2 = 2$$

$$\lambda_1 = 1: \begin{bmatrix} 15 & -12 \\ 20 & -16 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left.\begin{matrix} 15x_1 - 12x_2 = 0 \\ 20x_1 - 16x_2 = 0 \end{matrix}\right\} \quad x_1 = \tfrac{4}{5}x_2 = t \qquad p_1 = \begin{bmatrix} t \\ \frac{5}{4}t \end{bmatrix}$$

$$\lambda_2 = 2: \begin{bmatrix} 16 & -12 \\ 20 & -15 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left.\begin{matrix} 16x_1 - 12x_2 = 0 \\ 20x_1 - 15x_2 = 0 \end{matrix}\right\} \quad x_1 = \tfrac{3}{4}x_2 = t \qquad p_2 = \begin{bmatrix} t \\ \frac{4}{3}t \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix} \Rightarrow P^{-1} = \frac{1}{1}\begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -10 & 8 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

⑩

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda-1 & -1 \\ 0 & -1 & \lambda-1 \end{vmatrix} = (\lambda-1)[(\lambda-1)^2-1] = (\lambda-1)(\lambda^2-2\lambda) = (\lambda-1)(\lambda-2)\lambda = 0$$

$\boxed{\text{1.5 pts}}$

$$\lambda_1 = 0, \quad \lambda_2 = 1, \quad \lambda_3 = 2$$

$$\lambda_1 = 0: \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \left.\begin{matrix} -x_1 = 0 \\ -x_2 - x_3 = 0 \end{matrix}\right\} \quad x_2 = -x_3 = t \qquad p_1 = \begin{bmatrix} 0 \\ t \\ -t \end{bmatrix}$$

$$\lambda_2 = 1: \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \left.\begin{matrix} -x_3 = 0 \\ -x_2 = 0 \end{matrix}\right\} \quad x_1 = t \qquad p_2 = \begin{bmatrix} t \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_3 = 2: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \left.\begin{matrix} x_1 = 0 \\ x_2 - x_3 = 0 \end{matrix}\right\} \quad x_2 = x_3 = t \qquad p_3 = \begin{bmatrix} 0 \\ t \\ t \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \Rightarrow P^{-1} = \frac{1}{2}\begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$