(12)

1.5 pts

$$|A| = \begin{vmatrix} 19 & -9 & -6 \\ 25 & -11 & -9 \\ 17 & -9 & -4 \end{vmatrix} = 19(44-81)+9(-100+153)-6(-225+187) = -703+477+228 = 2$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-19 & 9 & 6 \\ -25 & \lambda+11 & 9 \\ -17 & 9 & \lambda+4 \end{vmatrix} = (\lambda-19)[(\lambda+11)(\lambda+4)-81] - 9[-25(\lambda+4)+153] + 6[-225+17(\lambda+11)] = 0$$

$$(\lambda-19)[\lambda^2+15\lambda-37] - 9[-25\lambda+53] + 6[17\lambda-38] = \lambda^3+15\lambda^2-37\lambda-19\lambda^2-285\lambda+703+225\lambda$$
$$-477+102\lambda-228$$

$$= \lambda^3-4\lambda^2+5\lambda-2 = (\lambda-1)(\lambda-1)(\lambda-2) = 0$$

$\lambda_1 = 2$, $\lambda_{2,3} = 1$

$$\lambda=1: \left[\begin{array}{ccc|c} -18 & 9 & 6 & 0 \\ -25 & 12 & 9 & 0 \\ -17 & 9 & 6 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} -18 & 9 & 6 & 0 \\ -7 & 3 & 3 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} -4 & 3 & 3 & 0 \\ 0 & 3 & -4 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] \Rightarrow \begin{matrix} -x_1+x_3=0 \\ 3x_2-4x_3=0 \end{matrix} \Bigg\} \begin{matrix} x_1 = x_3 = t \\ x_2 = \frac{4}{3}t \end{matrix}$$

Since the matrix is 3x3, and there is only one eigenvector for the eigenvalue $\lambda=1$, the matrix is not diagonalizable

(13)

1.5 pts

$$\det(\lambda I - A) = \begin{vmatrix} \lambda+1 & -4 & 2 \\ 3 & \lambda-4 & 0 \\ 3 & -1 & \lambda-3 \end{vmatrix} = (\lambda+1)[(\lambda-4)(\lambda-3)] + 4[3(\lambda-3)] + 2[-3-3(\lambda-4)] = 0$$

$$(\lambda+1)(\lambda-4)(\lambda-3) + 12\lambda-36-6\lambda+18 = (\lambda+1)(\lambda-4)(\lambda-3)+6\lambda-18 = (\lambda-3)[(\lambda+1)(\lambda-4)+6] = (\lambda-3)(\lambda^2-3\lambda+2)$$

$$= (\lambda-3)(\lambda-2)(\lambda-1) = 0 \qquad \lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3$$

$$\lambda_1 = 1: \begin{bmatrix} 2 & -4 & 2 \\ 3 & -3 & 0 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} x_1-2x_2+x_3=0 \\ x_1-x_2=0 \\ 3x_1-x_2-2x_3=0 \end{matrix} \Bigg\} x_1 = x_2 = x_3 = t \qquad p_1 = \begin{bmatrix} t \\ t \\ t \end{bmatrix}$$

$$\lambda_2 = 2: \begin{bmatrix} 3 & -4 & 2 \\ 3 & -2 & 0 \\ 3 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} 3x_1-4x_2+2x_3=0 \\ 3x_1-2x_2=0 \\ 3x_1-x_2-x_3=0 \end{matrix} \Bigg\} \begin{matrix} x_1 = \frac{2}{3}x_2 = t \\ x_3 = \frac{3}{2}x_1 \end{matrix} \qquad p_2 = \begin{bmatrix} t \\ \frac{3}{2}t \\ \frac{3}{2}t \end{bmatrix}$$

$$\lambda_3 = 3: \begin{bmatrix} 4 & -4 & 2 \\ 3 & -1 & 0 \\ 3 & -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} 2x_1-2x_2+x_3=0 \\ 3x_1-x_2=0 \end{matrix} \Bigg\} \begin{matrix} x_1 = \frac{1}{3}x_2 = t \\ x_3 = 4x_1 \end{matrix} \qquad p_3 = \begin{bmatrix} t \\ 3t \\ 4t \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 3 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 3 & -2 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -5 & 3 \\ 0 & 1 & 0 & -1 & 3 & -2 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 4 \end{bmatrix} \Rightarrow P^{-1} = \begin{bmatrix} 3 & -5 & 3 \\ -1 & 3 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 3 & -5 & 3 \\ -1 & 3 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -5 & 3 \\ -2 & 6 & -4 \\ 0 & -3 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$