# Math 20
# The Assignment Problem

1. You work as a sales manager for a toy manufacturer, and you currently have three salespeople on the road meeting buyers. Your salespeople are in Austin, TX; Boston, MA; and Chicago, IL. You want them to fly to three other cities: Denver, CO; Edmonton, Alberta; and Fargo, ND. The table below shows the cost of airplane tickets in dollars between these cities.

| From \ To | Denver | Edmonton | Fargo |
|---|---|---|---|
| Austin | 250 | 400 | 350 |
| Boston | 400 | 600 | 350 |
| Chicago | 200 | 400 | 250 |

   Where should you send each of your salespeople in order to minimize airfare?

2. A construction company has four large bulldozers located at four different garages. The bulldozers are to be moved to four different construction sites. The distances in miles between the bulldozers and the construction sites are given below.

| Bulldozer \ Site | A | B | C | D |
|---|---|---|---|---|
| 1 | 90 | 75 | 75 | 80 |
| 2 | 35 | 85 | 55 | 65 |
| 3 | 125 | 95 | 90 | 105 |
| 4 | 45 | 110 | 95 | 115 |

   How should the bulldozers be moved to the construction sites in order to minimize the total distance traveled?

3. Find an optimal assignment and corresponding cost for the following cost matrix using the Hungarian method.

$$\begin{bmatrix} 3 & -2 & 0 & 1 \\ 5 & 3 & -3 & 4 \\ 2 & 7 & 5 & 3 \\ 5 & -2 & 0 & 1 \end{bmatrix}$$

4. **The Bride-Groom Problem:** A marriage broker has four female clients and five male clients who desire to be married. She ranks the possible matchings between her clients on a scale of zero to ten; zero for the poorest match and ten for the best match. Her rankings are given in the following table.

| Bride \ Groom | Jacob | Michael | Joshua | Matthew | Andrew |
|---|---|---|---|---|---|
| Emily | 7 | 4 | 7 | 3 | 10 |
| Emma | 5 | 9 | 3 | 8 | 7 |
| Madison | 3 | 5 | 6 | 2 | 9 |
| Hannah | 6 | 5 | 0 | 4 | 8 |

   How should she match her clients in order to maximize the sum of the rankings of the matches? (Note that you will need to take two creative steps in order to solve this problem using the Hungarian method.)

## The Hungarian Method

The following algorithm applies the above theorem to a given $n \times n$ cost matrix to find an optimal assignment.

**Step 1.** Subtract the smallest entry in each row from all the entries of its row.

**Step 2.** Subtract the smallest entry in each column from all the entries of its column.

**Step 3.** Draw lines through appropriate rows and columns so that all the zero entries of the cost matrix are covered and the *minimum* number of such lines is used.

**Step 4.** *Test for Optimality:* (i) If the minimum number of covering lines is $n$, an optimal assignment of zeros is possible and we are finished. (ii) If the minimum number of covering lines is less than $n$, an optimal assignment of zeros is not yet possible. In that case, proceed to Step 5.

**Step 5.** Determine the smallest entry not covered by any line. Subtract this entry from each uncovered row, and then add it to each covered column. Return to Step 3.