

## Math 20

### Linear Programming and the Simplex Method

Unless otherwise noted, each of the following problems should be solved by hand, either using geometric techniques or the simplex method.

1. Maximize  $f = x - 4y$  subject to

$$\begin{aligned}x + 2y &\leq 4 \\x + 6y &\leq 8 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

2. Minimize  $f = -2x + y$  subject to

$$\begin{aligned}2x + y &\leq 440 \\4x + y &\leq 680 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

3. A city has \$1,800,000 to purchase cars. Two cars, the Arrow and the Gazelle, are under consideration, costing \$12,000 and \$15,000, respectively. The estimated annual maintenance cost on the Arrow is \$400 and on the Gazelle is \$300. The city will allocate \$40,000 for the total annual maintenance of these cars. The Arrow gets 28 miles per gallon and the Gazelle gets 25 miles per gallon. The city wants to maximize the “gasoline efficiency number” of this group of cars. For  $x$  Arrows and  $y$  Gazelles, this number would be  $28x + 25y$ . How many of each model should be purchased?
4. An industrial furniture company manufactures desks, cabinets, and chairs. These items involve metal, wood, and plastic. The following table gives the amounts that go into each product (in convenient units) and the profit on each item.

	Metal	Wood	Plastic	Profit
Desk	3	4	2	\$16
Cabinet	6	1	1	\$16
Chair	1	2	2	\$6

If the company has 800 units of metal, 400 units of wood, and 100 units of plastic available, how should it allocate these resources in order to maximize total profit?

5. Whizbang Widgets Inc produces three models of widgets, the wacky widget, the wonky widget, and the wicked widget. Producing a wacky widget requires 3 pounds of plastic and 6 minutes of labor. Producing a wonky widget requires 4 pounds of plastic and 3 minutes of labor. Producing a wicked widget requires 5 pounds of plastic and 2 minutes of labor. The profit for each wacky widget is \$2, the profit for each wonky widget is \$1.50, and the profit for each wicked widget is \$1.75. If 1000 pounds of plastic and 20 hours of labor are available each day, how many widgets of each model should Whizbang Widgets Inc. produce in order to maximize profits?
6. Donald desires to design a breakfast of corn flakes and milk that is as economical as possible. On the basis of what he eats during his other meals, he decides that his breakfast should supply him with at least 9 grams of protein, at least  $\frac{1}{3}$  the recommended daily allowance (RDA) of vitamin D, and at least  $\frac{1}{4}$  the RDA of calcium. He finds the following nutrition information on the milk and corn flake containers.

	Milk ( $\frac{1}{2}$ cup)	Corn Flakes (1 ounce)
Cost	7.5 cents	5.0 cents
Protein	4 grams	2 grams
Vitamin D	$\frac{1}{8}$ of RDA	$\frac{1}{10}$ of RDA
Calcium	$\frac{1}{6}$ of RDA	None

In order not to have his mixture too soggy or too dry, Donald decides to limit himself to mixtures that contain between 1 and 3 ounces of corn flakes per cup of milk, inclusive. Set up a linear programming problem to answer the question, What quantities of milk and corn flakes should Donald use to minimize the cost of his breakfast? Then solve this problem using Mathematica's `Minimize` command.