Math 20 Spring 2005
Midterm 2 Review Guide

1 Topics

The second midterm will cover §2.2, §2.3, §2.6, §2.8, §2.9, §4.9, §5.1–§5.3, §5.6, and §6.6 in our textbook, *Linear Algebra and Its Applications*, third edition. (That's everything since the first midterm up through §6.6 inclusive.) In particular, you will be responsible for the following topics.

§2.2 The Inverse of a Matrix

- Given two matrices, to verify that they are inverses.
- To simplify expressions involving sums, differences, scalar products, matrix products, transpositions, and *inverses* of matrices.
- To identify a given matrix as an elementary matrix and find its inverse.
- To illustrate how one can use elementary matrices to find the inverse of a matrix. (Theorem 7)
- To use row reduction to find the inverse of a matrix.

§2.3 Characterizations of Invertible Matrices

- To know and be able to apply the Invertible Matrix Theorem. (See the full Invertible Matrix Theorem later in this review guide.)

§2.6 The Leontief Input-Output Model

- To model a simple economy with the Leontief input-output model, finding the production levels for each of several sectors needed to satisfy both intermediate and final demand from those sectors.
- To analyze the entries of the matrix $(I - C)^{-1}$, where C is the consumption matrix in a Leontief input-output model.

§2.8 Subspace of $\mathbb{R}^n$

- To determine algebraically or geometrically if a given set of vectors is a subspace of $\mathbb{R}^n$.
- To determine whether a given vector is in the column space or null space of a given matrix.
- To determine if a given set of vectors is a basis for $\mathbb{R}^n$.
- To find a basis for the column space or null space of a given matrix.

§2.9 Dimension and Rank

- To find the coordinates of a given vector relative to a given basis.
- To find the dimension of a given subspace.
- To find the rank of a matrix (that is, the dimension of its columns space) or the dimension of its null space.

- To apply Theorem 15 to determine if a given set of vectors is a basis for a given subspace.
- To interpret and apply the additional Invertible Matrix Theorem statements lists on page 179.

§4.9 Applications to Markov Chains

- To find the transition matrix for a Markov chain given a verbal description of the Markov chain.
- To find and interpret future state vectors in a Markov chain given the transition matrix of the Markov chain and an initial state vector.
- To find and interpret the steady state vector of a Markov chain given the transition matrix of the Markov chain.

§5.1 Eigenvectors and Eigenvalues

- To determine geometrically the eigenvalues and eigenvectors of a linear transformation that is relatively easy to visualize, such as a reflection, rotation, projection, etc.
- To find a basis for the eigenspace of a given eigenvalue for a given matrix.
- To efficiently find the eigenvalues (and their multiplicities) of a triangular matrix.

§5.2 The Characteristic Equation

- To find the determinant of a 2×2 or 3×3 matrix.
- To apply the properties of determinants listed in Theorem 3.
- To find the characteristic polynomial of a given 2×2 or 3×3 or triangular matrix.
- To find the eigenvalues (and their multiplicities) of a given 2×2 or triangular matrix.

§5.3 Diagonalization

- To efficiently compute powers of a diagonal matrix.
- To efficiently compute powers of a diagonalized matrix.
- To use Theorem 5 to diagonalize a matrix A by finding an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.
- To use Theorems 6 and 7 to determine if a given matrix is diagonalizable.

§5.6 Discrete Dynamical Systems

- To find the transition matrix for a discrete dynamical system given a verbal description of the system.
- To express the kth state vector of a discrete dynamical system as a linear combination of the eigenvectors of the system's transition matrix. (Equation 2 on page 343)
- To determine the long-term behavior of a discrete dynamical system given the eigenvalues and eigenvectors of its transition matrix.

§6.6 Applications to Linear Models

- To find the equation of the least-squares line that best fits a small set of data with one independent variable and one dependent variable.
- To describe the model that produces a least-squares fit of a set of data with one independent variable and one dependent variable by a function of a certain form.
- To describe the model that produces a least-squares fit of a set of data with two independent variables and one dependent variable by a function of a certain form.

2 Suggested Exercises

- §2.2 #15, 19, 21, 23, 29, 31
- §2.3 #13, 15, 17, 19, 21, 27
- §2.6 #1, 3, 5, 7
- §2.8 #11, 13, 17, 25, 27, 29, 31, 33
- §2.9 #5, 7, 11, 13, 15, 19, 21, 23
- §4.9 #1, 3, 11, 13
- §5.1 #9, 15, 17, 19, 25, 29, 31
- §5.2 #7, 9, 17, 19, 23, 25
- §5.3 #5, 9, 11, 13, 27, 31
- §5.6 #3, 5, 17(a, b)
- §6.6 #3, 7(a), 9

3 The Invertible Matrix Theorem

Let A be a square $n \times n$ matrix. Then the following statements are equivalent. That is, for a given A, the statements are either all true or all false.

(a) A is an invertible matrix.

(b) A is row equivalent to the $n \times n$ identity matrix.

(c) A has n pivot positions.

(d) The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

(e) The columns of A form a linearly independent set.

(f) The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.

(g) The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.

(h) The columns of A span $\mathbb{R}^n$.

(i) The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.

(j) There is an $n \times n$ matrix C such that $CA = I$.

(k) There is an $n \times n$ matrix D such that $AD = I$.

(l) A^T is an invertible matrix.

(m) The columns of A form a basis of $\mathbb{R}^n$.

(n) The column space of A is $\mathbb{R}^n$.

(o) The dimension of the column space of A is n.

(p) The rank of A is n.

(q) The null space of A equals $\{\mathbf{0}\}$.

(r) The dimension of the null space of A is 0.

(s) The number 0 is not an eigenvalue of A.

(t) The determinant of A is not zero.