# Math 20
## §1.4 The Matrix Equation $A\mathbf{x} = \mathbf{b}$

**Question 1:** Note that $x_1 = 11$, $x_2 = 2$, $x_3 = 1$ is a solution to the following system of linear equations.

$$\begin{array}{rrrrrrr} x_1 & + & 4x_2 & + & 2x_3 & = & 21 \\ 2x_1 & - & x_2 & - & 5x_3 & = & 15 \end{array}$$

Which of the following might not be true?

1. The vector $\mathbf{x} = \begin{bmatrix} 11 \\ 2 \\ 1 \end{bmatrix}$ is a solution to the matrix equation

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & -1 & -5 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 21 \\ 15 \end{bmatrix}$$

.

2. $b_1 = 21$, $b_2 = 15$ is a solution to the vector equation

$$b_1 \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + b_2 \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \\ 1 \end{bmatrix}$$

.

3. $x_1 = 11$, $x_2 = 2$, $x_3 = 1$ is a solution to the vector equation

$$x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ -1 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 21 \\ 15 \end{bmatrix}$$

.

**Question 2:** Note that $x_1 = 11$, $x_2 = 2$, $x_3 = 1$ is a solution to the following system of linear equations.

$$\begin{array}{rrrrrrr} x_1 & + & 4x_2 & + & 2x_3 & = & 21 \\ 2x_1 & - & x_2 & - & 5x_3 & = & 15 \end{array}$$

Which of the following is not an implication of this fact?

1. The matrix equation $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & -1 & -5 \end{bmatrix}$, has a solution for any $b$ in $\mathbb{R}^2$.

2. The vector $\begin{bmatrix} 21 \\ 15 \end{bmatrix}$ is a linear combination of the columns of the matrix $\begin{bmatrix} 1 & 4 & 2 \\ 2 & -1 & -5 \end{bmatrix}$.

3. The vector $\begin{bmatrix} 21 \\ 15 \end{bmatrix}$ is in the subset of $\mathbb{R}^2$ spanned by the vectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$.

**Question 3:** Suppose that the system of linear equations whose augmented matrix is $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{bmatrix}$ is consistent. Let $A$ be the matrix with columns $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. Let $m$ be the length of the vector $\mathbf{b}$. Which of the following must be true?

1. The matrix $A$ has a pivot position on every row.

2. The vector $\mathbf{b}$ is in the subset of $\mathbf{R}^m$ spanned by the columns of $A$.

3. The columns of $A$ span $\mathbf{R}^m$.

4. The columns of $A$ span $\mathbf{R}^3$.

**Question 4:** Let $A = \begin{bmatrix} 1 & 5 \\ -2 & -13 \\ 3 & -3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Suppose that the matrix equation $A\mathbf{x} = \mathbf{b}$ is consistent. Which of the following is not true?

1. The entries of $\mathbf{b}$ must satisfy the equation

$$15b_1 + 6b_2 - b_3 = 0.$$

2. The vector $\mathbf{b}$ lies in the plane consisting of all linear combinations of the columns of $A$.

3. The echelon form of $A$ has a row of zeros.

4. The vector $\mathbf{b}$ can be any vector in $\mathbb{R}^3$.